

# HECKE CHARACTERS OF SINGULAR DRINFELD MODULES

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**The  $j$ -invariant  $j$  of a Drinfeld module of rank 2 on  $\mathbf{F}_q[T]$  over  $C$  determine an isomorphism class of Drinfeld modules over  $C$ . But for singular Drinfeld modules the pair  $(j, \chi)$  of a singular  $j$ -invariant  $j$  and an algebraic Hecke character  $\chi$  represent an  $H$ -isomorphism class of singular Drinfeld modules, where  $H$  is the Hilbert class field of certain imaginary quadratic function field.**

**0. Introduction.** In the theory of elliptic curves (or more generally, abelian varieties) with complex multiplication, the Hecke characters play some important roles, such as the classification of isogeny classes of elliptic curves and the study of zeta functions. In the theory of Drinfeld modules, Gross introduced the notion of algebraic Hecke characters [Gr1]. In this note we restrict ourselves to the Hecke characters arising from the singular Drinfeld modules of rank 2 on  $A = \mathbf{F}_q[T]$ , and see the correspondences between isogeny classes or isomorphism classes of singular Drinfeld modules and Hecke characters.

We fix the following notations throughout this paper:

$\mathbf{F}_q$  : finite field of  $q$ -elements

$A = \mathbf{F}_q[T]$ ,  $k = \mathbf{F}_q(T)$

$K$  = quadratic extension of  $k$  where  $\infty$  does not split

$L^s$  = separable closure of a field  $L$

$\overline{L}$  = algebraic closure of a field  $L$ .

**1. Hecke Characters and Frobenius morphism.** Let  $L$  be an  $A$ -field. In this note by a Drinfeld module over  $L$  we always mean a Drinfeld module of rank 2 on  $A$ . Thus a Drinfeld module  $\phi$  is completely determined by

$$\phi_T(X) = TX + gX^q + \Delta X^{q^2}$$