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L^p-BOUNDEDNESS OF THE HILBERT TRANSFORM AND MAXIMAL FUNCTION ALONG FLAT CURVES IN $\mathbb{R}^n$

SARAH N. ZIESLER

We consider the Hilbert transform and maximal function associated to a curve $\Gamma(t) = (t, \gamma_2(t), \dots, \gamma_n(t))$ in $\mathbb{R}^n$. It is well-known that for a plane convex curve $\Gamma(t) = (t, \gamma(t))$ these operators are bounded on L^p, $1 < p < \infty$, if γ' doubles. We give an n-dimensional analogue, $n \geq 2$, of this result.

1. Introduction. Let $\Gamma : \mathbb{R} \longrightarrow \mathbb{R}^n$ be a curve in $\mathbb{R}^n$, $n \geq 2$, with $\Gamma(0) = 0$. We define the associated Hilbert transform, $\mathcal{H}_\Gamma$ and maximal function $\mathcal{M}_\Gamma$ by

$$\mathcal{H}_\Gamma f(x) = \mathrm{p.\,v.} \int_{-\infty}^{\infty} f(x - \Gamma(t)) \frac{dt}{t}$$

and

$$\mathcal{M}_\Gamma f(x) = \sup_{r>0} \frac{1}{r} \int_0^r |f(x - \Gamma(t))| \, dt,$$

respectively. We use p. v. to indicate that we are taking a principal value integral.

There has been considerable interest in finding conditions on Γ which give $L^2(\mathbb{R}^n)$-boundedness or $L^p(\mathbb{R}^n)$-boundedness, $1 < p < \infty$, of $\mathcal{H}_\Gamma$ and $\mathcal{M}_\Gamma$, when Γ is permitted to be flat (i.e. vanish to infinite order) at the origin; the case of well-curved Γ was dealt with in the 1970's, see for example [**7**].

The aim of this paper is to give an n-dimensional analogue of the following well-known theorem for plane curves.

THEOREM 1.1. [**1**]. *Let $\Gamma : \mathbb{R} \longrightarrow \mathbb{R}^2, \Gamma(t) = (t, \gamma(t))$ be a convex curve such that $\gamma \in C^2(0, \infty)$ is either even or odd and $\gamma(0) = \gamma'(0) = 0$. Suppose that $\exists 1 < \lambda < \infty$ such that $\forall t \in (0, \infty)$*

$$\gamma'(\lambda t) \geq 2\gamma'(t). \tag{1}$$