

ESSENTIAL LAMINATIONS AND HAKEN NORMAL FORM

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We show that if a 3-manifold M contains an essential lamination, then for any triangulation of M there exists an essential lamination which is in Haken normal form with respect to that triangulation.

0. Introduction. The notion of (Haken) normal form w.r.t. a triangulation of a 3-manifold traces back to Kneser's work in the 1930's on surfaces in 3-manifolds. Haken studied it extensively in the 1960's, and showed [8] how to use it to create finite algorithms for the determination of various properties of embedded surfaces. This has since culminated, in the work of Jaco and Oertel [10], in an algorithm to determine if an irreducible 3-manifold is a Haken manifold, i.e., if it contains a 2-sided incompressible surface.

In [7] a generalization of the incompressible surface, the essential lamination, was introduced. There it was shown that a 3-manifold M containing an essential lamination has some of the same desirable properties of a 3-manifold containing an incompressible surface, the most notable property being that M has universal cover $\mathbb{R}^3$. Since then, it has also been shown [6] that, in some sense, 'most' 3-manifolds contain essential laminations.

The purpose of this paper is to prove a Haken normal form result for essential laminations.

The reader is referred to [7] for definitions and basic properties concerning essential laminations. In this paper the word 'lamination' will mean a lamination which is carried by a branched surface, i.e., it has 'air' between its leaves. Since we will ultimately be interested only in the existence of an essential lamination with certain properties, this additional restriction will cause no difficulties; we can 'blow air' between the leaves of a foliation (see [7]) to obtain a lamination in our sense.