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A DIOPHANTINE EQUATION CONCERNING FINITE GROUPS

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In this paper we prove that all solutions (y, m, n) of the equation $3^m - 2y^n = \pm 1$, $y, m, n \in \mathbb{N}$, $y > 1, m > 1, n > 1$, satisfy $y < 10^{6\cdot 10^9}$, $m < 1,4 \cdot 10^{15}$ and $n < 1,2 \cdot 10^5$.

1. Introduction. Let $\mathbb{Z}, \mathbb{N}, \mathbb{P}, \mathbb{Q}$ be the sets of integers, positive integers, odd primes and rational numbers respectively. In [**2**], Cresenzo considered the solutions (p, q, m, n, δ) of the equation

$$(1)\, p^m - 2q^n = \delta,\ p, q \in \mathbb{P},\ m, n \in \mathbb{N},\ m > 1, n > 1, \delta \in \{-1, 1\},$$

which is concerned with finite groups. He claimed that if $(p, q, m, n, \delta) \neq (239, 13, 2, 4, -1)$, then $(m, n, \delta) = (2, 2, -1)$. However, we notice that (1) has another solution $(p, q, m, n, \delta) = (3, 11, 5, 2, 1)$ with$(m, n, \delta) \neq (2, 2, -1)$. Thus it can be seen that the above result is not correct. If we follow the proof of Cresenzo, we can argue as follows. The above result is deduced from the following lemma:

LEMMA A ([**2**, Lemma 1]). *Suppose that $q \in \mathbb{P}$ and $x, m, n \in \mathbb{N}$. If*

$$x^m - 2q^n = \pm 1,\ x > 1,\ m > 1,\ n > 1, \tag{2}$$

then m is a power of 2. Furthermore, the sign of the term ± 1 must be negative.

Notice that if $2 \nmid xm$, then from (2) we get

$$2q^k = x \mp 1$$

for some $k \in \mathbb{N}$ with $k < n$. Now there are two cases:

$$x = \begin{cases} 3, & \text{if } k = 0, \\ 2q^k \pm 1, & \text{if } k > 0. \end{cases} \tag{3}$$