ON DEDEKIND'S FUNCTION $\eta(\tau)$

WILHELM FISCHER

1. Introduction. A transformation of the form

$$\tau' = \frac{a\tau + b}{c\tau + d}\ , \tag{1.1}$$

where a, b, c, d are rational integers satisfying

$$\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - cb = 1\ , \tag{1.2}$$

is called a *modular transformation.* Without loss of generality we may assume $c \geq 0$. A function $f(\tau)$, analytic in the upper halfplane $\mathfrak{J}(\tau) > 0$, and satisfying the functional equation

$$f(\tau) = (c\tau + d)^k\ f\left(\frac{a\tau + b}{c\tau + d}\right), \tag{1.3}$$

is called a *modular form of dimension k.* An example of a modular form is the discriminant

$$\Delta(\tau) = \exp\{2\pi i\tau\} \prod_{m=1}^{\omega} (1 - \exp\{2\pi i m\tau\})^{24}\ , \tag{1.4}$$

which is of dimension -12; that is, it satisfies the equation*

$$\Delta(\tau') = (c\tau + d)^{12}\Delta(\tau)\ . \tag{1.5}$$

An important role in the theory of modular functions is played by the function

$$\eta(\tau) = \exp\left\{\frac{\pi i\tau}{12}\right\} \prod_{m=1}^{\infty} (1 - \exp\{2\pi i m\tau\})\ , \tag{1.6}$$

Received June 12, 1950.

*Cf. Hurwitz [6]; however, he gives this formula only in homogeneous coordinates.

Pacific J. Math. 1 (1951), 83-95.