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MINIMAL SURFACES WITH CATENOID ENDS

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A substantive part of the recent activity in the field of minimal surface theory has been the construction of new complete minimal surfaces immersed in $\mathbb{R}^3$. One approach in constructing new examples is to increase the genus of known minimal surfaces. In this paper, we do precisely this for certain minimal surfaces of finite total curvature whose ends are asymptotic to catenoids. We prove existence of surfaces of positive genus based on those in genus zero, with the feature that these higher genus examples maintain all the symmetry of their genus-zero counterparts. In these proofs we use the conjugate minimal surface construction and the maximum principle for minimal surfaces.

1. Introduction

In the last century, O. Bonnet, and later H. A. Schwarz, were the first to study the associate family of a minimal surface ([**Ni2**], [**Scz**]). More recently, A. Schoen, H. Karcher, and others have used properties of the associate family to develop a method for constructing periodic minimal surfaces ([**Ka1**], [**Ka2**], [**Ka3**], [**Ka4**], [**Kr**]). This method uses the particular member of the associate family known as the conjugate surface, and is referred to, by Karcher, as the *Conjugate Plateau Construction.* W. H. Meeks III suggested using this construction to study non-periodic examples ([**Me**]).

This construction is used to prove the results here. The technique begins by considering the boundary contour of the conjugate of a fundamental piece of the surface. The contour can be described as partially unbounded boundary data over an unbounded convex domain. We extend results of J. C. C. Nitsche [**Ni1**] and Jenkins and Serrin [**JeSe**] to this setting and obtain the existence of a unique minimal surface with this given boundary. The existence of the original surface can then be argued.

Our main results concern the existence of immersed finite-total-curvature minimal surfaces with embedded catenoid ends and genus greater than zero:

1) For each $n \geq 3$, there exists an n-oid of genus 1 that maintains all the symmetry of the genus-0 n-oid (see Figures 1.1 and 1.2).