PACIFIC JOURNAL OF MATHEMATICS
Vol. 172, No. 1, 1996

COMMUTING CO-COMMUTING SQUARES AND FINITE DIMENSIONAL KAC ALGEBRAS

TAKASHI SANO

A relationship between finite dimensional Kac algebras and specified commuting co-commuting squares is discussed. The Majid's bicrossproduct Kac algebra is explained in our context.

1. Introduction.

The theory of Kac algebras (Hopf algebras) has been drawing considerable attention (see [**6**] for the reference), and in fact many intensive studies have been made recently. ([**1**, **18**, **19**, **34**, **35**, **36**], etc.) On the other hand, the announcement by A.Ocneanu ([**20**, **21**]) brought us a new aspect in the theory of Kac algebras : it is his claim (proved in [**4**, **17**] and also [**28**]) that, for an irreducible inclusion of factors $M \supset N$ with finite index and depth $=$ 2, M is described as the crossed product algebra of N by an outer action of a finite dimensional Kac algebra. Hence, we investigate Kac algebras from the Jones index theoretical point of view.

The purpose of this paper is to find a finite dimensional Kac algebra via the index theory : let $L \supset K$ be an irreducible inclusion of factors with finite index. Suppose that, for an intermediate subfactor M, both inclusions $L \supset M$ and $M \supset K$ are of depth 2. Although the inclusion $L \supset K$ does not always satisfy the depth 2 condition, it can be proved that this pair is of depth 2 if these factors L, M, K, and another intermediate subfactor N form a commuting co-commuting square. Details will be explained in §2 after recalling basic facts on commuting co-commuting squares. Another criterion for the inclusion $L \supset K$ to be of depth 2 is also obtained. Examples are given in §3.

The author would like to express his sincere gratitude to Professor Shigeru Yamagami for helpful advice (in fact the present work was motivated by [**33**]) and to Professor Hideki Kosaki for fruitful discussions and constant encouragement. He is grateful to the referee for many useful comments.