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FINITELY GENERATED COHOMOLOGY HOPF ALGEBRAS AND TORSION

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H-spaces X whose $\operatorname{mod} p$ cohomology is finitely generated as an algebra are studied. Even generators of infinite height lie in degrees $2p^j$ for $j \geq 0$. If $H^*(X; \mathbb{Z}_p)$ is not finite dimensional, then $H^*(X; \mathbb{Z})$ must have p torsion of all orders if X is two connected.

Introduction.

In this note we begin a study of H-spaces whose cohomology mod p is not finite dimensional, but is finitely generated as an algebra. We study these H-spaces by studying the structure of their Borel decompositions. From the Borel structure theorem, if the mod p cohomology of an H-space is nonfinite, either there are an infinite number of algebra generators or there are elements of infinite height. We prove the following:

Theorem A. *Let A be a mod p cohomology Hopf algebra admitting an action of the Steenrod algebra. If A is finitely generated as an algebra, then the generators of infinite height lie in degrees of the form $2p^j$, for $j \geq 0$.*

In the next theorem we show that nonfinite H-spaces must have unbounded p-torsion in their cohomology.

Theorem B. *Let X be a two-connected H-space and suppose $p^r H^*(X; \mathbb{Z})$ is torsion-free for some $r > 0$. Then if $H^*(X;\mathbb{Z}_p)$ is finitely generated as an algebra, then it is finite dimensional.*

Theorem C. *Let X be a two-connected H-space with $H^*(X; \mathbb{Z}_p)$ finitely generated as an algebra, but not finite dimensional. Then $\beta_1 Q H^{\text{even}}(X; \mathbb{Z}_p) \neq 0$, where β_1 is the first cohomology Bockstein.*

Theorem D. *Let X be a two-connected H-space with $H^*(X; \mathbb{Z}_p)$ finitely-generated as an algebra, but not finite dimensional. Then $H^*(X; \mathbb{Z})$ has p-torsion of all orders.*

Corollary E. *Let X be a homotopy associative H-space with $H^*(X; \mathbb{Z}_p)$ finitely generated as an algebra and primitively generated. If p is an odd*