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MOON HYPERSURFACES AND SOME RELATED EXISTENCE RESULTS OF CAPILLARY HYPERSURFACES WITHOUT GRAVITY AND OF ROTATIONAL SYMMETRY

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Let $\Omega_*(R)$ be a domain in $\mathbb{R}^n$ bounded by two spherical caps Σ_1 and Σ_2 of respective radii $\frac{n-1}{n}$ and R, with $\frac{n-1}{n} < R < 1$. (cf. Figure 1 for $n = 3$). We consider the vertical cylinder Z over $\partial\Omega_*(R)$ and seek a hypersurface $u_R(x_1, \dots, x_n)$ over $\Omega_*(R)$ of constant mean curvature $H \equiv 1$ which meets Z in the angle π (vertically downward) over $\Sigma_1(R)$ and the angle 0 (vertically upward) over $\Sigma_2(R)$; intuitively and essentially, this amounts to seeking a solution to the problem

$$\begin{cases} \operatorname{div} Tu_R = & n \\ \nu \cdot Tu_R = & \begin{cases} -1 & \text{on } \Sigma_1(R) \\ 1 & \text{on } \Sigma_2(R), \end{cases} \end{cases} \tag{0.1}$$

ν being outward unit normal.

0. Introduction.

In view of the shape of the base domain $\Omega_*(R)$, we shall, as in [**FG**] for $n = 2$, refer to $\Omega_*(R)$ as *n-dimensional moon domains* and as in [**F2**], refer to the solution of (0.1) as *moon (hyper)-surfaces*. Such a moon surface ($n = 2$) is chosen to majorize the gradient of solution $u(x)$ of

$$\operatorname{div} Tu = 2 \tag{0.2}$$

in $B_R, R_0^{(2)} < R < 1$, with $R_0^{(2)} = 0.565406\dots$ being the unique value of R for which $\Sigma_1(R)$ passes through the center of the circle including $\Sigma_2(R)$. This enables us to show the existence of apriori gradient bounds for solution of the equation (0.2) in $B_R, R_0^{(2)} < R < 1$, in [**FG**].

0.1. We note that, an integration of (0.1) over the section $\Omega_*(R)$ yields

$$|\Sigma_2(R)| - |\Sigma_1(R)| = n|\Omega_*(R)|. \tag{0.3}$$

Thus, the condition (0.3) is necessary for existence of the moon hypersurfaces u_R.