PACIFIC JOURNAL OF MATHEMATICS
Vol. 172, No. 2, 1996

ON NORMALITY OF THE CLOSURE OF A GENERIC TORUS ORBIT IN G/P

Romuald Dabrowski

In this paper we consider generic orbits for the action of a maximal torus T in a connected semisimple algebraic group G on the generalized flag variety G/P, where P is a parabolic subgroup of G containing T. The union of all generic T-orbits is an open dense (possibly proper, if P is not a Borel subgroup) subset of the intersection of the big cells in G/P. We prove that the closure of a generic T-orbit in G/P is a normal equivariant T-embedding (whose fan we explicitely describe). Moreover, the closures of any two generic T-orbits are isomorphic as equivariant T-embeddings.

1. Introduction.

Let G be a connected semisimple algebraic group over an algebraically closed field k of arbitrary characteristic. As usual, let B^+ denote a fixed Borel subgroup of G, T a maximal torus in B^+, $\Gamma(T)$ the character group of T, B the opposite to B^+, Φ the corresponding root system in an euclidian space $(E, (\ , \ ))$, Φ_+ the set of positive roots relative to B^+, Δ the set of simple roots in Φ_+, s_α the reflection about the linear subspace of E perpendicular to root α, W the Weyl group of Φ generated by the reflections $s_\alpha, \alpha \in \Phi_+$ (W can also be naturally identified with $N_G(T)/T$), and R the root lattice in E.

Let P be a fixed parabolic subgroup containing B. Let Δ_P be the set of simple roots α such that $s_\alpha \in W_P = N_P(T)/T$. Then the map $P \to \Delta_P$ is a bijection between the set of all parabolic subgroups containing B and the power set of Δ (see e.g. [**B**, Proposition 14.18]). We denote by S^P the subsemigroup of the root lattice generated by all positive roots which are not sums of simple roots in Δ_P.

We will be concerned with T-orbits of points in the projective variety G/P. Let λ be an integral dominant weight (with respect to Φ_+) whose stabilizer in W is W_P Then λ extends to a character of P (we will also call it λ), inducing a line bundle $\mathcal{L}^\lambda$ on G/P. We let $V(\lambda)$ denote the Weyl G-module

$$H^0(G/P, \mathcal{L}^\lambda) = \{f \in k[G] | f(xy) = \lambda^{-1}(y) f(x) \text{ for all } x \in G, y \in P\}$$