PACIFIC JOURNAL OF MATHEMATICS
Vol. 173, No. 2, 1996

THE WEIL REPRESENTATION AND GAUSS SUMS

ANTONIA BLUHER

We use the Weil representation to evaluate certain Gauss sums over a local field, up to ± 1. Also we construct a cocycle on $\mathrm{Sp}(2m, \mathbb{R})$ with a simple formula on the maximal compact torus and we show how to lift homomorphisms $j : \mathrm{Sp}(2n, \mathbb{R}) \to \mathrm{Sp}(2m, \mathbb{R})$ to the double covers of these groups.

1. Introduction.

Let F be a self-dual local field of char $\neq 2$, for instance $F = \mathbb{R}$, $\mathbb{C}$, or a finite extension of $\mathbb{Q}_p$. For most of the paper we will assume $F \neq \mathbb{C}$. Let χ be a nontrivial additive character of F. Then all additive characters of F have the form $\lambda\chi$ for some $\lambda \in F$, where $\lambda\chi(t) = \chi(\lambda t)$. We consider the following unitary operators on $L^2(F^m)$:

$$(\mathbf{a}(A)\Phi)(X) = |\det A|_F^{1/2}\, \Phi(XA) \qquad \text{for } A \in \mathrm{GL}_m(F), \tag{1.1}$$

$$\mathbf{n}(B)\Phi(X) = \chi(XBX^T/2)\, \Phi(X) \qquad \text{for } B = B^T \in \mathrm{M}_m(F), \tag{1.2}$$

(1.3)
$$(\mathcal{F}_j\Phi)(X) = \int_{F^j} \Phi(Y_1, \dots, Y_j, X_{j+1}, \dots, X_m)\, \chi(X_1Y_1 + \cdots + X_jY_j)\, dY,$$

$$(\iota(t)\Phi)(X) = t\Phi(X), \qquad t \in \mathbf{T} = \{z \in \mathbb{C} \,|\, z\bar{z} = 1\}. \tag{1.4}$$

Here Φ is a nice function in $L^2(F^m)$ (to be precise, Φ belongs to the Schwartz space $\mathcal{S}(F^m)$), dY is an additive Haar measure on F^j normalized so that $\mathcal{F}_j^2 = \mathbf{a}(\mathrm{diag}\{-I_j, I_{m-j}\})$ for $0 \leq j \leq m$, and $|a|_F$ for $a \in F$ is the modulus function, determined by $d(ya) = |a|_F\, dy$ for a Haar measure dy on $(F, +)$. All our vectors are row vectors. We will usually suppress the symbol ι; that is, identify t with $\iota(t)$ for $t \in \mathbf{T}$. Let $\mathrm{Mp} = \mathrm{Mp}(F^m)$ be the topological group generated by all the above operators. We call this the metaplectic group. This group is independent of χ since $\lambda\chi(XBX^T/2) = \chi(X(\lambda B)X^T/2)$ and $\mathbf{a}(\begin{pmatrix} \lambda I_j & \\ & I_{m-j} \end{pmatrix})\mathcal{F}_{j,\chi} = \mathcal{F}_{j,\lambda\chi}$, where we have added a subscript to the Fourier