PACIFIC JOURNAL OF MATHEMATICS
Vol. 174, No. 1, 1996

THE SCHWARTZ SPACE OF A GENERAL SEMISIMPLE LIE GROUP V: SCHWARTZ CLASS WAVE PACKETS

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Suppose G is a connected semisimple Lie group. Then the tempered spectrum of G consists of families of representations induced unitarily from cuspidal parabolic subgroups. In the case that G has finite center, Harish-Chandra used Eisenstein integrals to construct wave packets of matrix coefficients for each series of tempered representations. He showed that these wave packets are Schwartz class functions and that each K-finite Schwartz function is a finite sum of wave packets. Thus he obtained a complete characterization of K-finite functions in the Schwartz space in terms of their Fourier transforms.

Now suppose that G has infinite center. Then every K-compact Schwartz function decomposes naturally as a finite sum of wave packets. A new feature of the infinite center case is that the wave packets into which it decomposes are not necessarily Schwartz class functions. This is because of interference between different series of representations when a principal series representation decomposes as a sum of limits of discrete series. There are matching conditions between the wave packets which are necessary in order that the sum be a Schwartz class function when the individual terms are not. In this paper it is shown that these matching conditions are also sufficient. This gives a complete characterization of K-compact functions in the Schwartz space in terms of their Fourier transforms.

1. Introduction.

Suppose G is a connected semisimple Lie group. Then the tempered spectrum of G consists of families of representations induced unitarily from cuspidal parabolic sub-groups. Each family is parameterized by the unitary characters of a Cartan subgroup. The Plancherel theorem expands Schwartz class functions on G in terms of the distribution characters of these tempered representations. Very roughly, for f in the Schwartz space $\mathcal{C}(G)$, we can write

$$f(x) = \sum_{H \in \mathrm{Car}(G)} f_H(x), x \in G \tag{1.1 a}$$