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CONVOLUTION AND LIMIT THEOREMS FOR CONDITIONALLY FREE RANDOM VARIABLES

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Dedicated to Professor Wilhelm von Waldenfels on the occasion of his 65th birthday

We introduce the notion of a conditionally free product and conditionally free convolution. We describe this convolution both from a combinatorial point of view, by showing its connection with the lattice of non-crossing partitions, and from an analytic point of view, by presenting the basic formula for its R-transform. We calculate explicitly the distributions of the conditionally free Gaussian and conditionally free Poisson distribution.

1. Introduction.

In [**BSp**], we introduced a generalization with respect to two states of the reduced free product of Voiculescu [**Voi1**, **VDN**] and gave some preliminary results on this concept. Here, we want to examine this notion more systematically, in particular, we want to investigate the corresponding convolution. We describe this convolution both from a combinatorial point of view – by showing its connection with the lattice of non-crossing partitions – and from an analytic point of view – by presenting the basic formula for its R-transform, which is the replacement of the classical Fourier-transform. We calculate explicitly the distributions of the corresponding Gaussian and Poisson law by a careful examination of the structure of the non-crossing partitions.

Instead of the terms "ψ-independence" and "ψ-product" of [**BSp**], we will use here the more precise expressions "conditionally free" and "conditionally free product", or just the abbreviation "c-free".

Let us start with a motivation for our concept of "c-freeness". Consider a group $G = *_{i\in I} G_i$ which is the free product of groups G_i $(i \in I)$, i.e. each element $g \neq e$ of G can be written uniquely in the form $g = g_1 \dots g_n$, where $e \neq g_j \in G_{i(j)}$ and $i(1) \neq i(2) \neq \cdots \neq i(n)$. To see the nature of this