BOUNDARY BEHAVIOR OF THE BERGMAN CURVATURE IN STRICTLY PSEUDOCONVEX POLYHEDRAL DOMAINS

KANG-TAE KIM AND JIYE YU

In this article, we present an explicit description of the boundary behavior of the holomorphic curvature of the Bergman metric of bounded strictly pseudoconvex polyhedral domains with piecewise C^2 smooth boundaries. Such domains arise as an intersection of domains with strongly pseudoconvex domains with C^2 smooth boundaries, creating normal singularities in the boundary. Our results in particular yield an optimal generalization of the well-known theorem of Klembeck, in terms of the boundary regularity. As an application, we demonstrate generalization of several theorems which were previously known only for the cases of everywhere C^{∞} (essentially) smooth boundaries.

1. Introduction.

Let D be a bounded domain in \mathbb{C}^n . Consider the space

$$\mathcal{H}^2(D):=\left\{f:D o\mathbb{C}\,|\,f\,\, ext{is holomorphic},\int_D|f|^2d\mu<\infty
ight\}$$

where $d\mu$ is the standard volume form of \mathbb{C}^n . This space is usually called the *Bergman space*. Equipped with the standard L^2 norm, it is a separable Hilbert space. Therefore, we choose an orthonormal basis $\{\varphi_j\}_{j=0}^{\infty}$ for the Bergman space. Then the *Bergman kernel function* $K: D \times D \to \mathbb{C}$ can be obtained by

$$K(z, \overline{\zeta}) := \sum_{j=1}^{\infty} \varphi_j(z) \overline{\varphi_j(\zeta)},$$

where $z, \zeta \in D$. This function gives rise to the well-known *Bergman metric* of D as follows:

$$\sum_{\alpha,\beta=1}^n g_{\alpha\bar\beta} dz^\alpha \otimes d\overline{z^\beta} := \sum_{\alpha,\beta=1}^n \frac{\partial^2 \log K(z,\bar z)}{\partial z^\alpha \partial \overline{z^\beta}} dz^\alpha \otimes d\overline{z^\beta}.$$

One of the important features of this metric is that it is one of the invariant Kähler metrics, in the sense that the biholomorphic mappings are isometries with respect to the Bergman metric.