

LOCAL AND GLOBAL PLURISUBHARMONIC DEFINING FUNCTIONS

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Let D be a bounded pseudoconvex domain with real-analytic boundary in $\mathbb{C}^2$ which satisfies a geometric condition on the set of points where the Levi form degenerates. If locally D has smooth defining functions plurisubharmonic on the boundary, then D has a global smooth defining function plurisubharmonic on the boundary.

1. Introduction.

In analysis on pseudoconvex domains one often encounters the problem of passing from local to global information. Here we consider this problem as it relates to defining functions plurisubharmonic on the boundary of certain pseudoconvex domains in $\mathbb{C}^2$, in the sense that at each boundary point the Levi form of the defining function is positive semi-definite on all vectors in $\mathbb{C}^2$, not just complex tangent vectors to the boundary (cf. [1]). In [3], Fornæss constructed a bounded pseudoconvex domain with real-analytic boundary in $\mathbb{C}^2$ which near each boundary point has a local defining function plurisubharmonic on the boundary, but which nevertheless has no global defining function plurisubharmonic on the boundary. This domain has the geometric property that the set of points where the Levi form degenerates is a curve which always points in the direction of the complex tangent space to the boundary. Our main result is that if a domain is linearly regular—which essentially requires that this geometric property does not hold—then one can pass from local to global defining functions plurisubharmonic on the boundary:

Theorem. *Let D be a linearly regular domain with real-analytic boundary in $\mathbb{C}^2$, and suppose that for each $p \in D$ there is a neighborhood U_p of p on which D has a smooth defining function which is plurisubharmonic on $\partial D \cap U_p$. Then D has a global smooth defining function plurisubharmonic on ∂D .*

The precise definition of linear regularity is given in the next section. For now we note that it has proved to be relevant to similar problems. This