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DIVERGENCE OF THE NORMALIZATION FOR REAL LAGRANGIAN SURFACES NEAR COMPLEX TANGENTS

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We study real Lagrangian analytic surfaces in $\mathbb{C}^2$ with a non-degenerate complex tangent. Webster proved that all such surfaces can be transformed into a quadratic surface by formal symplectic transformations of $\mathbb{C}^2$. We show that there is a certain dense set of real Lagrangian surfaces which cannot be transformed into the quadratic surface by any holomorphic (convergent) transformation of $\mathbb{C}^2$. The divergence is contributed by the parabolic character of a pair of involutions generated by the real Lagrangian surfaces.

1. Introduction.

We consider a real analytic surface M in $\mathbb{C}^2$. Let $\omega = dz \wedge dp$ be the holomorphic symplectic 2-form on $\mathbb{C}^2$. M is a *real Lagrangian* surface if

$$\text{Re}\ \omega|_M = 0. \tag{1.1}$$

The real Lagrangian surfaces were initially studied by S.M. Webster [**10**]. It was known that all totally real and real Lagrangian analytic submanifolds are equivalent under holomorphic symplectic transformations. When M has a non-degenerate complex tangent, Webster proved that under formal symplectic transformations, M can be transformed into the quadratic surface

$$Q : p = 2z\overline{z} + \overline{z}^2. \tag{1.2}$$

Furthermore, M can be transformed into Q by holomorphic transformations of $\mathbb{C}^2$ if and only if they are equivalent through holomorphic symplectic transformations [**10**]. The purpose of this paper is to show that there exist real Lagrangian surfaces such that the above normal form cannot be realized by any holomorphic (convergent) transformation.

In [**7**], J.K. Moser and S.M. Webster systematically investigated the holomorphic invariant theory of real surfaces in $\mathbb{C}^2$, where a pair of involutions intrinsically attached to the complex tangents plays an important role. We shall see that the divergence for the normalization of the real Lagrangian surfaces is contributed by the parabolic character of the pair of involutions.