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ALGEBRAIC EQUIVALENCE OF LOCALLY NORMAL REPRESENTATIONS

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It will be shown that (i) the absolute value of every locally normal linear functional is again locally normal; (ii) two locally normal representations π_1 and π_2 of $\mathscr{A}$ generate isomorphic von Neumann algebras $\mathscr{M}(\pi_1)$ and $\mathscr{M}(\pi_2)$ if and only if there exists an automorphism σ of $\mathscr{A}$ such that $\pi_1 \circ \sigma$ and π_2 are quasi-equivalent, provided that either $\mathscr{M}(\pi_1)$ or $\mathscr{M}(\pi_2)$ is σ-finite.

This paper is motivated by a recent work [6] of R. Haag. R. V. Kadison and D. Kastler. As they mentioned, the recent progress in mathematical physics has made a precise analysis of representations of a C^*-algebra furnished with a net of von Neumann algebras a growing necessity.

In the first half of this paper, we shall show that the space of all locally normal linear functionals of a C^*-algebra with a net of von Neumann algebras is a closed invariant subspace of the conjugate space in the sense of [14], which will imply that the absolute value of a locally normal linear functional is locally normal too.

The last half of this paper will be devoted to extending a result of Powers [11] for UHF algebra to a C^*-algebra $\mathscr{A}$ with a proper sequential type I_∞ funnel. Namely it will be shown that two locally normal representations π_1 and π_2 of the C^*-algebra $\mathscr{A}$ generate isomorphic von Neumann algebras if and only if they are connected by an automorphism of $\mathscr{A}$. This is proven under the assumption that one of the generated von Neumann algebras is σ-finite.

2. The locally normal conjugate space of a C^*-algebra with a net of von Neumann algebras. Let $\mathscr{A}$ be a C^*-algebra. Suppose a system $\mathfrak{F} = (\mathscr{A}_\alpha\}$ of C^*-subalgebras of $\mathscr{A}$ indexed by a directed set $\{\alpha\}$ is given such that:

(i) $\mathscr{A}_\alpha$ is a von Neumann subalgebra of $\mathscr{A}_\beta$ if $\alpha \leqq \beta$;

(ii) $\bigcup_\alpha \mathscr{A}_\alpha$ is dense in $\mathscr{A}$ with respect to the norm topology.

The system $\mathfrak{F} = \{\mathscr{A}_\alpha\}$ is called a *net* (in $\mathscr{A}$) of von Neumann algebras and each $\mathscr{A}_\alpha$ is called *local subalgebra* of $\mathscr{A}$

DEFINITION 1. A continuous linear functional φ (resp. representation π) of $\mathscr{A}$ is said to be *locally normal* if φ (resp. π) is σ-weakly continuous on each local subalgebra $\mathscr{A}_\alpha$.