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THE SPECTRUM OF CERTAIN LOWER TRIANGULAR MATRICES AS OPERATORS ON THE l_p SPACES

JAMES D. STAFNEY

In this paper we compute the spectrum of the lower triangular matrices $A_a = (c_{m,n})$, where $c_{m,n} = (n+1)^a/(m+1)^{a+1}$, $m \geqq n \geqq 0$, a is real and the corresponding operator on l_p is bounded (see 4.1). This result and other lemmas are used to determine the spectrum of lower triangular matrices $p(n)/q(m)$, $m \geqq n \geqq 0$ as operators on l_p where p is a monic polynomial of degree a, q is a monic polynomial of degree $a+1$ and $q(m) \neq 0$ for $m = 0, 1, \cdots$. The spectrum is the diagonal together with the set $C_{a-p} - 1_{+1}$ when $a - p^{-1} + 1 > 0$, where $C_b = \{\lambda: |\lambda - (2b)^{-1}| \leqq (2b)^{-1}\}$ (see 4.3).

Our initial interest in this problem stems from conversations with Prof. Charles J. A. Halberg, Jr., who had conjectured and partially proved the conclusions of Theorem 4.1 for an operator equivalent to the special case $a = 1$.

1. Preliminaries.

In this section we set down the general notation and prove some preliminary lemmas.

General notation 1.1. Let X be a complex normed linear space. The norm is denoted $\| \ \|$ or $\| \ \|_X$ if it can be confused with another norm. The normed algebra of bounded linear operators on X is denoted $O(X)$. For $T \in O(X)$, $\mathrm{sp}(T)$ or $\mathrm{sp}(T, X)$ denotes the spectrum; that is, all complex numbers λ such that $\lambda - T$ does not have an inverse in $O(X)$. If $T \in O(X)$ and K is a subspace of X such that $TK \subset K$, we let $T|K$ denote the operator in $O(K)$ obtained by restricting T to K.

For $1 \leqq p \leqq \infty$ (p will always denote a number in this range) l_p is the usual normed linear space of complex p-summable sequences $x = (x_0, x_1, \cdots)$. We will be concerned with complex matrices $A = (a_{m,n})$, $0 \leqq m, n < \infty$. It is well known that there is a one-to-one correspondence between $O(l_p)$ and a class of matrices and that this correspondence is an algebra isomorphism. In this paper we will not disinguish between an operator in $O(l_p)$ and its corresponding matrix; in particular, we will speak of matrices as elements in $O(l_p)$. A lower triangular matrix $A = (a_{m,n})$ is a matrix such that $a_{m,n} = 0$ if $m < n$. $\mathscr{L}_p$ will denote the lower triangular matrices in $O(l_p)$. The set $\{a_{n,n}: n = 0, 1, \cdots\}$ is denoted $d(A)$. A sequence A_k of matrices is said to converge to A entrywise if the m, nth entry of A_k converges