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ON SATURATED FORMATIONS OF SOLVABLE LIE ALGEBRAS

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The concepts of formations, $\mathscr{F}$-projectors and $\mathscr{F}$-normalizers have all been developed for solvable Lie algebras. In this note, for each saturated formation $\mathscr{F}$ of solvable Lie algebras, the class $\mathscr{T}(\mathscr{F})$ of solvable Lie algebras L in which each $\mathscr{F}$-normalizer of L is an $\mathscr{F}$-projector is considered. This is the natural generalization of the Lie algebra analogue to SC groups which were first investigated by R. Carter. It is shown that $\mathscr{T}(\mathscr{F})$ is a formation. Then some properties of $\mathscr{F}$-normalizers of $L \in \mathscr{T}(\mathscr{F})$ are considered.

All Lie algebras considered here are solvable and finite dimensional over a field F. $\mathscr{F}$ will always denote a saturated formation of solvable Lie algebras and L will be a solvable Lie algebra. $N(L)$ is the nil-radical of L and $\Phi(L)$ is the Frattini subalgebra of L. For definitions and properties of all these concepts see [3], [4], and [9]. For SC groups see [6].

We begin with a general lemma.

LEMMA 1. *Let N be an ideal of L and D/N be an $\mathscr{F}$-normalizer of L/N. Then there exists an $\mathscr{F}$-normalizer E of L such that $E + N = D$.*

Proof. Let L be a minimal counterexample and we may assume that N is a minimal ideal of L. If $D/N = L/N$, then any $\mathscr{F}$-normalizer of L has the desired property, hence we may suppose that $D/N \subset L/N$. Suppose first that N is $\mathscr{F}$-central in L. Let $N^*/N = N(L/N)$ and $C = C_L(N)$. Then $N(L) = N^* \cap C$. Let M/N be a maximal $\mathscr{F}$-critical subalgebra of L/N such that D/N is an $\mathscr{F}$-normalizer of M/N. Now either M is $\mathscr{F}$-critical in L or M complements a chief factor of L between N^* and $N(L)$. In the first case, by induction, there exists an $\mathscr{F}$-normalizer E of M such that $E + N = D$ and E is also an $\mathscr{F}$-normalizer in L. In the second case, $L/C \in \mathscr{F}$ and $C + N^*/C$ is operator isomorphic to $N^*/N^* \cap C = N^*/N(L)$. Hence each chief factor of L between N^* and $N(L)$ is $\mathscr{F}$-central which contradicts M being $\mathscr{F}$-abnormal.

Now suppose that N is $\mathscr{F}$-eccentric and assume $N \subseteqq \Phi(L)$. Let M/N be as in the above paragraph. Again, by induction, there exists an $\mathscr{F}$-normalizer E of M such that $E + N = D$. But $N \subseteqq \Phi(L)$ yields that M is $\mathscr{F}$-critical in L using Theorem 2.5 of [4]. Hence E is an