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GROUPS OF MATRICES ACTING ON DISTRIBUTION SPACES

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Let E be a locally convex space of temperate distributions on the n-dimensional Euclidean space R^n, and G aclosed subgroup of $\mathrm{Gl}(n, R)$, the general linear group over R^n. An attempt is made to identify those distributions which can be approximated in E by linear combinations of distributions of the form $u(Ax+b)$, where u is a fixed element of E, A varies over G, and b varies over R^n. A cancellation theorem is proved; this then allows the support of the Fourier transform of any annihilator of the set of distributions of the form $u(Ax+b)$ to be localized. This in turn is used to obtain approximation results.

1. Notation. Throughout, R^n denotes n-dimensional Euclidean space. The character group of R^n is again R^n, the identification being made in such a way that multiplicative factors in the Fourier inversion formula are eliminated. The Haar measure on R^n is denoted by dx.

We denote by $C_c^\infty(U)$ the space of indefinitely differentiable functions on R^n which have compact support inside the open set U in R^n. $S(R^n)$ is the space of rapidly decreasing indefinitely differentiable functions on R^n. The space of all Schwartz distributions on R^n is designated by $D'(R^n)$, and its subspace consisting of temperate distributions is denoted by $S'(R^n)$.

$\mathrm{Gl}(n, R)$ is the general linear group over R^n. The determinant of an element A in $\mathrm{Gl}(n, R)$ is written $\det A$, and A' denotes the adjoint matrix of A.

Now, consider a fixed element A in $\mathrm{Gl}(n, R)$. Then it is easy to see that the function

$$x \longrightarrow \phi(Ax) \quad (x \in R^n)$$

belongs to $C_c^\infty(R^n)$ whenever ϕ does. We write ϕ^A for this function. This definition is extended to include all distributions by making use of the adjoint of the map which carries ϕ onto ϕ^A. More precisely, if u is a distribution, then we define u^A to be the unique distribution which satisfies

$$\langle \phi, u^A\rangle = |\det A^{-1}|\langle \phi^{A^{-1}}, u\rangle \qquad (\phi \in C_c^\infty(R^n)) .$$

The translate of a distribution u by an element b in R^n is defined in the usual fashion, and denoted by u_b. We write u_b^A for $(u_b)^A$.