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# ON HOMOGENEOUS ALGEBRAS

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**If $A$ is an algebra over a field $K$ let Aut $(A)$ denote the group of algebra automorphisms of $A$. Then $A$ is said to be extremely homogeneous if Aut $(A)$ act transitively on $A \backslash \{0\}$. Also $A$ is said to be homogeneous if Aut $(A)$ acts transitively on the one-dimensional subspaces of $A$. The purpose of this paper is to investigate some of the basic properties of homogeneous algebras. In particular, the alternative homogeneous algebras and the homogeneous algebras of dimension 2 are classified.**

**All algebras are assumed to be finite dimensional and not necessarily associative.**

We now include a brief historical account of this topic. The concept of an extremely homogeneous algebra arose from a particular problem in the structure of certain finite $p$-groups as studied by Boen, Rothaus and Thompson [1]. Extremely homogeneous algebras have been investigated by Kostrikin [4]. Homogeneous algebras over finite fields other than $GF(2)$ have been investigated by Shult [6], [7], and his results completed the work on the related $p$-groups. The case of homogeneous algebras over $GF(2)$ was considered by Gross [3]. Swierczkowski classified all real homogeneous Lie algebras [9] and finally Dyokovic classified all real homogeneous algebras [2]. A homogeneous algebra $A$ is said to be nontrivial if $A^2 \neq 0$ and $\dim A > 1$. The author has shown that there are no nontrivial homogeneous algebras over an algebraically closed field [8].

The paper is divided into five sections: arbitrary homogeneous algebras, alternative homogeneous algebras, power-associative homogeneous algebras, homogeneous quasi-division algebras and finally homogeneous algebras of dimension 2.

I. Arbitrary homogeneous algebras. Let $A$ be an arbitrary algebra over a field $K$. Then left multiplication by a fixed element $a \in A$ induces a linear map on $A$ which is denoted by $L_a$. Similarly right multiplication by $a$ induces a linear map on $A$ denoted by $R_a$. We do not distinguish between the map $L_a$ and its matrix representation relative to some fixed basis. By End $(A)$ we indicate the vector space of all linear maps on $A$. By $L$ we indicate the subspace of End $(A)$ consisting of all $L_x$ as $x$ runs through $A$ and similarly for $R$. An algebra $A$ is said to be nonzero if $A^2 \neq 0$.

THEOREM 1. *Let $A$ be a nonzero homogeneous algebra over a*