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COMPACTLY COGENERATED LCA GROUPS

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In this paper we seek to describe and investigate a class of LCA groups which appropriately generalizes the class of finitely cogenerated abelian groups. Of three possible generalizing classes we finally choose one, which we refer to as the class of compactly cogenerated LCA groups, as being the most suitable. It turns out that this class is considerably more complicated than the corresponding class of compactly generated LCA groups. We give various criteria for an LCA group to be a member of this class, and we describe several important subclasses. As a result of our investigations we show that a divisible LCA group which is indecomposable is either compact and connected, or else is isomorphic to the group of real numbers, a quasicyclic group, or a p-adic number group.

1. Introduction. Within the category of abelian groups the finitely generated groups play an important rôle. The natural generalization of this class within the category of locally compact abelian (LCA) groups is the class of compactly generated groups, about which much detailed information is available (see, for example, [3, § 9] including the well-known structure theorem [3, 9.8]. Dual to the class of finitely generated groups within the category of abelian groups is the class of finitely cogenerated groups (see [2, pp. 109–111]). It is the purpose of this paper to investigate possible generalizations of this class within the category of LCA groups.

Throughout, all groups will be assumed to be LCA Hausdorff topological groups. The LCA groups which we mention frequently are the circle T, the real numbers R, the integers Z, the cyclic groups $Z(n)$, the rationals Q, the quasicyclic groups $Z(p^\infty)$, the p-adic integers J_p and the p-adic numbers F_p. Precise definitions of all these groups may be found in [3]. Topological isomorphism will be denoted by "$\cong$".

Let us recall the definition of a finitely cogenerated abelian group. A subset S of an abelian group G is called a *system of cogenerators* of G if for every abelian group H and homomorphism $f: G \to H$ we have $\ker(f) \cap S \subseteq \{0\} \Rightarrow f$ is a monomorphism. An abelian group is then called *finitely cogenerated* if it contains a finite system of cogenerators. It is shown in [2, Theorem 25.1] that G is finitely cogenerated if and only if the subgroups of G satisfy the minimum condition, in which case G is the direct sum of finitely many cocyclic