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JOINS OF DOUBLE COSET SPACES

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The double cosets of a group by a subgroup and the irreducible complex characters of a finite group have a structure which can be studied by means of hypergroups (alias convos, probability groups, and Pasch geometries). An external operation on these structures called the "join" is studied. Decomposition theorems are established in both the general hypergroup (Pasch geometry) case and the weighted hypergroup (probability group) case.

1. Geometries and Joins.

DEFINITION 1.1. A Pasch Geometry (see (3)) is a triple (A, Δ_A, e) where A is a set, $e \in A$ and $\Delta_A \subseteqq A \times A \times A$ satisfying:

(I) $\forall a \in A$, $\exists$ unique $b \in A$ with $(a, b, e) \in \Delta_A$. Denote b by $a^\sharp$.

(II) $e^\sharp = e$ and $(a^\sharp)^\sharp = a \forall a \in A$.

(III) $(a, b, c) \in \Delta_A \Rightarrow (b, c, a) \in \Delta_A$.

(IV) (Pasch's axiom) $(a_1, a_2, a_3), (a_1, a_4, a_5) \in \Delta_A \Rightarrow \exists a_6 \in A$ with $(a_6, a_4^\sharp, a_2), (a_6, a_5, a_3^\sharp) \in \Delta_A$.

We may often write A for (A, Δ_A, e) and Δ for Δ_A if the context is clear. Also we use the word "geometry" for Pasch Geometry.

For $B \subseteqq A$, we denote $B\backslash\{e\}$ by B^*. If $B \subseteqq A$, B is called a *subgeometry* of A, and we write $B < A$, iff (i) $e \in B$ and (ii) $(b_1, b_2, a) \in \Delta_A$ with $b_1, b_2 \in B$ implies that $a \in B$. If A and C are geometries, a map $f: A \to C$ is called a *geometry morphism* iff $f(e) = e$ and $(x, y, z) \in \Delta_A$ implies $(f(x), f(y), f(z)) \in \Delta_C$.

1.2. Definition of $A//B$. If A is a geometry and $B<A$, it can be shown that the following relation $\sim$ is an equivalence relation on A:

$$x \sim y \text{ iff } \exists b_1, b_2 \in B, a \in A \quad \text{with} \quad (x, b_1, a^\sharp), (a, y^\sharp, b_2) \in \Delta .$$

If the equivalence class of $a \in A$ is denoted by $[a]$ and we let $A//B$ denote $\{[a]: a \in A\}$, then in fact $(A//B, \Delta, [e])$ is a geometry where $([x], [y], [z]) \in \Delta$ iff $\exists x' \in [x], y' \in [y], z' \in [z]$ with $(x', y', z') \in \Delta_A$.

If A is a group (with $\Delta = \{x, y, z): xyz = 1\}$) and B is a subgroup of A, then $A//B$ defines a geometry structure on the set of $B - B$ double cosets of A.

The following result is routine from the definitions.

PROPOSITION 1.3. *Suppose that A and B are geometries and $f: A \to B$ is a geometry morphism. Let $K_f = \{a \ni A: f(a) = e\}$ and*