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SPACES WITHOUT REMOTE POINTS

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All spaces considered are completely regular and X^* denotes $\beta X - X$. The point $x \in X^*$ is called *a remote point of* X if $x \notin \mathrm{Cl}_{\beta X} A$ for each nowhere dense subset A of X. If $y \in Y$, then the space Y is said to be *extremally disconnected at* y if $y \notin \overline{U} \cap \overline{V}$ whenever U and V are disjoint open sets. In this paper we construct two noncompact σ-compact spaces X, one locally compact and one nowhere locally compact, such that X has no remote points, and in fact such that βX is not extremally disconnected at any point.

Our examples were motivated by the following results from [**6**]:

(1) *X has remote points if X has countable π-weight, in particular if X is separable and first countable, and is not pseudocompact*, [**6**, 1.5]; see also [**7**] for an earlier consistency result, and [**1**] for a more general result.

(2) *βX is extremally disconnected at each remote point of X*, [**6**, 5.2]. Via the observation that

(3) *if Y is dense in Z, and $y \in Y$, then Y is extremally disconnected at y iff Z is extremally disconnected at y*,

these results and the following imply a nonhomogeneity result, which applies for example to the rationals and the Sorgenfrey line

(4) *if X is a nowhere locally compact nonpseudocompact space which has a remote point and if $\{x \in X$: X is not extremally disconnected at $x\}$ is dense in X, e.g. if X is first countable, then X^* is not homogeneous because X^* is extremally disconnected at some but not at all points.*

(This is a special case of Frolík's theorem that X^* is not homogeneous if X is not pseudocompact, [**8**]. The proof of Frolík's theorem does not yield a simple "because" as in (4). X is called nowhere locally compact if no point of X has a compact neighborhood, or, equivalently, if X^* is dense in βX.)

In this paper we produce two closely related examples which show that the condition on the π-weight cannot be omitted altogether in (1), thus answering a question of [**6**].

Our two examples are rather big: they have cellularity at least ω_3. This suggests the question of whether every nonpseudocompact separable space has a remote point. (This would generalize (1).) It follows from a construction in [7] that the answer is affirmative under CH.