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# DEGREE–ONE MAPS ONTO LENS SPACES

CLAUDE HAYAT–LEGRAND, SHICHENG WANG AND HEINER ZIESCHANG

**The paper deals with the question about the existence or non-existence of a degree-one map of a closed orientable 3-manifold $M$ to some lens space. The answer to this question is determined by the cyclic decomposition of $H_1(M)$, except when $H_1(M)$ contains an even number of direct factors isomorphic to $\mathbf{Z}_{2^k}$. In this case one has to calculate the linking matrix of $M$ to get the answer. For every $n$ even, we give a Seifert manifold $M_n$ with $H_1(M_n) \cong \mathbf{Z}_n \oplus \mathbf{Z}_n$ that does not admit a degree-one map to $L(n,m)$ for any $m$.**

## 1. Introduction.

Motivated by articles by Y. Rong [**10**] also Y. Rong and S. Wang [**11**], we consider the question whether a given 3-manifold $M$ admits a degree-one map to a lens space $L(n,m)$ or not. For every $n$ a geometric criterium is given (Theorem 2.2) by the linking pairing $\operatorname{Tor} H_1(M) \otimes \operatorname{Tor} H_1(M) \to \mathbf{Q}/\mathbf{Z}, (\alpha,\beta) \mapsto \alpha \odot \beta$. Moreover it has the advantage of being quite simple and useful for the geometric construction of examples with a negative answer. Because it is a pairing on abelian group it is easy to prove (Theorem 2.10) that the answer to the question about the existence of a degree-one map to a $L(n,m)$ is determined by the cyclic decomposition of $H_1(M)$ except for the case where $n = 2^k\bar{n}$, $k > 0$, $\bar{n}$ odd and $H_1(M)$ contains an even number of direct factors isomorphic to $\mathbf{Z}_{2^k}$. In the end one has to calculate the linking pairing only in this case.

For the case $L(n,m) = L(2,1) = \mathbf{P}^3$ the often used method depends on the existence of a generator $\xi \in H^1(L(2,1),\mathbf{Z}_2)$ with the property $\xi \cup \xi \cup \xi \neq 0$. If $n$ is odd or a multiple of 4 (see for example Remark 2.8 (c)) there is no element $\xi \in H^1(L(n,m),\mathbf{Z}_n)$ with $\xi^3 \neq 0$, but there is a generator $\mu \in H_1(L(n,m))$ with linking number $\mu \odot \mu \neq 0$ [**16**, 14.7.3 (c)]. This is the ingredient for Theorem 2.2. If there is a degree-one map $M \to L(n,m)$ then $H_1(M)$ has a direct factor isomorphic to $\mathbf{Z}_n$, see [**2**], [**16**, 14.2.6]. Exploring the geometric description of the linking number we construct for every even $n$ a Seifert manifold $M_n$ with $H_1(M_n) = \mathbf{Z}_n \oplus \mathbf{Z}_n$ such that there is no degree-one map from $M_n$ to a lens space $L(n,m)$. If $n = 2$ the Seifert manifold $M_2$