Journal of the Institute of Polytechnics,
Osaka City University,
Vol. 1, No. 1, Series A

Relations between Homotopy and Homology II

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§ 1. Introduction

After the publication of my last paper, "Relations between homotopy and homology I" [1], I had an opportunity of reading H. Hopf's papers [2], [3]. In those papers I found that both his method and results were somewhat similar to mine. Let us compare the geometrical part, the grouptheoretical part being excluded. Hopf only thought of "Homotopie-ränder" and dealt with the subgroup Π_0 defined by the notion of free homotopy, while I dealt with three homomorphisms, namely, homotopy boundary, homotopy relativisation and covering, as a part of the exact homomorphism sequence of homotopy groups. For the lack of the idea of free homotopy in my method, I could not deal with the low dimensional case. The difference of methods leads to different results. Taking Hopf's idea into consideration, we can see the difference between Hopf's group $\mathfrak{G}^n$ and my "simple group" Θ^n, and its geometrical nature; moreover the homology theory of a complex will be found to be reduced, in a sense, completely into the homotopy theory.

§ 2. Hopf's free homotopy group

Let $\{\alpha_i\}$ be the generators of the q-dimensional homotopy group $\pi_q(R)$ of a locally contractible topological space R and $\{\xi_j\}$ be the generators of the fundamental group $\pi_1(R)$. The Whitehead's product $\xi_j \cdot \alpha_i$ is also an element of $\pi_q(R)$. We denote by Γ_q the normal subgroup generated by all the elements $\{\xi_j \cdot \alpha_i\}$. The free homotopy group $\tilde{\pi}_q(R)$ is defined as the factor group $\pi_q(R)/\Gamma_q(R)$, i. e. if we add new defining relations $\{\xi_j \cdot \alpha_i = 1\}$ to the relations of π_q, then we get the group $\tilde{\pi}_q$. We denote by u the natural homomorphism from π_q to $\tilde{\pi}_q$. It is easily seen from the definition that for any elements $\alpha \in \pi_q$, $\xi \in \pi_1$, $\xi \cdot \alpha = 1$ in $\tilde{\pi}_q$. If R is q-simple in the sense of Eilenberg [4], then it is clearly $\pi_q = \tilde{\pi}_q$.

Sometimes we take a relative operator domain $\pi_1(L)$, where L is a closed connected subset of R. Replacing $\pi_1(R)$ with $\pi_1(L)$ we get a