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On the Uniform Topology of Bicompactifications.

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In this note we shall characterise the uniform space, which is a uniform subspace of its bicompactification $\beta(R)$ or $w(R)$ [1], by introducing the notion of u-normality. Then we shall study some properties of u-normal spaces. We denote by R a uniform space having uniformity $\{\mathfrak{M}_x \mid \mathfrak{X}\}$.

Let A and B be subsets of R and $A \frown B = \phi$. When there exists $\mathfrak{M}_x \in \{\mathfrak{M}_x\}$ such that $S(A, \mathfrak{M}_x) \frown B = \phi$, we say that A and B are *u-separated.* It is easy to see that A and B are u-separated, when and only when there exists a uniformly continuous function φ such that

$$\begin{aligned} \varphi(a) &= 0 \ (a \in A), \\ & \qquad\qquad 0 \leqq \varphi(a) \leqq 1. \\ \varphi(a) &= 1 \ (a \in B), \end{aligned}$$

A uniform space R is called a *Čech u-normal space,* when any disjoint completely closed sets [2] of R are u-separated, and is called a *u-normal space,* when any disjoint closed sets of R are u-separated.

Lemma 1. *In order that R is Čech u-normal, it is necessary and sufficient that every bounded continuous functions of R are uniformly continuous.*

Proof. Since for any disjoint completely closed sets F and G, there exists a bounded continuous function φ such that

$$\begin{aligned} \varphi(a) &= 0 \ (a \in F), \\ & \qquad\qquad 0 \leqq \varphi(a) \leqq 1. \\ \varphi(a) &= 1 \ (a \in G), \end{aligned}$$

the sufficiency of the condition is obvious.

Conversely let R be Čech u-normal, then any finite open covering $\mathfrak{N} = \{N_i \mid i = 1, \dots, n\}$ is an element of $\{\mathfrak{M}_x\}$, if every N_i^c are completely closed.[3]

For put $N_i^c = F_i$, then $\bigcap_{i=1}^{n} F_i = \phi$ or $F_1 \frown (F_2 \frown \cdots \frown F_n) = \phi$, where F_1 and $F_2 \frown \cdots \frown F_n$ are completely closed. Hence there exists