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STRUCTURE OF HEREDITARY ORDERS OVER LOCAL RINGS

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Let R be a noetherian integral domain and K its quotient field, and Σ a semi-simple K-algebra with finite degree over K. If Λ is a subring in Σ which is finitely generated R-module and $\Lambda K=\Sigma$, then we call it an order. If Λ is a hereditary ring, we call it a hereditary order (briefly h-order).

This order was defined in [1], and the author has substantially studied properties of h-orders in [5], and shown that we may restrict ourselves to the case where R is a Dedekind domain, and Σ is a central simple K-algebra.

In this note, we shall obtain further results when R is a discrete rank one valuation ring. Let R be such a ring, and Ω a maximal order with radical $\mathfrak{N}$, and $\Omega/\mathfrak{N}=\Delta_n$; Δ division ring. Then we shall show the following results: 1) Every h-order contains minimal h-orders Λ such that $\Lambda/N(\Lambda)\approx\Sigma\oplus\Delta$, where $N(\Lambda)$ is the radical of Λ, (Section 3); 2) The length of maximal chains for h-order is equal to n, and we can decide all chains which pass a given h-order, (Section 5); 3) For two h-orders Γ_1 and Γ_2 they are isomorphic if and only if they are of same form, (see definition in Section 4); 4) The number of h-orders in a nonminimal h-order is finite if and only if $R/\mathfrak{p}$ is a finite field, where $\mathfrak{p}$ is a maximal ideal in R, (Section 6).

In order to obtain those results we shall use a fundamental property of maximal two-sided ideals in Λ; $\{\mathfrak{M}, \mathfrak{N}^{-1}\mathfrak{M}\mathfrak{N}, \mathfrak{N}^{-2}\mathfrak{M}\mathfrak{N}^2, \cdots, \mathfrak{N}^{-r+1}\mathfrak{M}\mathfrak{N}^{r-1}\}$ gives a complete set of maximal two-sided ideals in Λ, where $\mathfrak{N}=N(\Lambda)$, (Section 2).

H. Higikata has also determined h-orders over local ring in [8] by direct computation and the author owes his suggestions to rewrite this paper, (Section 6). However, in this note we shall decide h-orders as a ring, namely by making use of properties of idempotent ideals and radical.

We only consider h-orders over local ring in this paper, except Section 1, and problems in the global case will be discussed in [7] and in a