Journal of the Institute of Polytechnics,
Osaka City University,
Vol. 5, No. 1, Series A

Meromorphic approximations on Riemann surfaces

By Kanenji SAKAKIHARA

(Received Feb. 1, 1954)

Let D, D' be compact domains of a Riemann surface R relative to R such that $\bar{D} \subset D'$ and D be enclosed by a finite number of closed Jordan curves. Let P be a finite point set contained in D, Q' be a selected set of the collection of compact components of $D'-\bar{D}$ relative to D', that is, any point of Q' is contained in one and only one element of the collection and conversely any element of the collection contains one and only one point of Q', and Q be a selected set of the collection of compact components of $R-\bar{D}$ relative to R. Obviously both Q' and Q are finite point sets. Then we have the following theorems:

THEOREM 1′. *There exists such a function as is meromorphic in D' and has its poles on P.*

THEOREM 2′. *Any function which is regular in a certain domain containing $\bar{D}$ is uniformly approximated on $\bar{D}$ by such a function as is meromorphic on D' and has its poles on Q'.*

THEOREM 3′. *Any functon which is meromorphic in a certain domain containing $\bar{D}$ and has its poles on P is uniformly approximated on $\bar{D}$ by such a function as is meromorphic in D' and has its poles on $P \cup Q'$.*

THEOREM 1. *There exists such a function as is meromorphic in R and has its poles on P.*

THEOREM 2. *Any function which is regular in a certain domain contiaining $\bar{D}$ is uniformly approximated on $\bar{D}$ by such a function as is meromorphic in R and has its poles on Q.*

THEOREM 3. *Any function which is meromorphic in a certain domain containing $\bar{D}$ and has its poles on P is uniformly approximated on $\bar{D}$ by such a function as is meromorphic in R and has its poles on $P \cup Q$.*

According to the method of Behenke and Stein[1], these theorems are easily derived by the following process[2]:

---

1) Behnke und Stein: Entwicklung analytischer Funktionen auf Riemannschen Flächen, Math. Ann. 120 (1948), pp. 430-461.

2) Theorem 1′ is trivial. Theorem 2′ is a modified one of a theorem in the above paper in which D is simply connected relative to D'.