Sairaiji, F.
Osaka J. Math.
37 (2000), 789–799

ON CONGRUENCES BETWEEN THE COEFFICIENTS OF TWO L-SERIES WHICH ARE RELATED TO A HYPERELLIPTIC CURVE OVER Q

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(Received February 1, 1999)

1. Introduction

Let $f(x)$ be a monic irreducible polynomial with rational integer coefficients and let p be a prime integer. Reducing the coefficients of $f(x)$ modulo p, we obtain the polynomial $f_p(x)$ with coefficients in $\mathbf{Z}/p\mathbf{Z}$. A rule of the factorization of $f_p(x)$ over $\mathbf{Z}/p\mathbf{Z}$ is called a reciprocity law for $f(x)$ (cf. Wyman [11]). For example, when $f(x)$ is of degree 2, a reciprocity law for $f(x)$ is given by the Legendre symbol (D_f/p) for the discriminant D_f of $f(x)$.

In the case that $f(x)$ is of degree 3, the minimal splitting field K of $f(x)$ over $\mathbf{Q}$ is the Galois extension generated by the coordinates of the two-division points of the elliptic curve $E : y^2 = f(x)$. A reciprocity law for $f(x)$ is given by the Legendre symbol (D_f/p) and the coefficients of the L-series of E over $\mathbf{Q}$, which is the Mellin transform of a modular form of weight two under the Taniyama-Shimura conjecture (the Wiles theorem). Furthermore, in the case that $f(x)$ is of degree 3 and $D_f < 0$, the inverse Mellin transform of the Artin L-function $L(\pi, K/\mathbf{Q}, s)$ attached to the two-dimensional irreducible representation π for the Galois group of K over $\mathbf{Q}$, is a modular form of weight one, by the Weil-Langlands theorem. Thus the Fourier coefficients of the modular form of weight one also gives a reciprocity law for $f(x)$.

In the latter case, we can associate two modular forms with E and the Galois extension generated by the coordinates of its two-division points. Koike [3] obtained congruences between the Fourier coefficients of two modular forms. His congruences describe the relation of the above two reciprocity laws. Naito [6] gave congruences between the coefficients of the L-series of E and those of an Artin L-series attached to the Galois extension generated by the coordinates of the three-division points of E.

In this paper we consider congruences modulo 2 between the coefficients of the L-series of the Jacobian variety of a hyperelliptic curve $y^2 = f(x)$ and those of an Artin L-series which is related to the Galois extension over $\mathbf{Q}$, generated by the coordinates of the two-division points of the same Jacobian variety.

Let $f(x)$ be a polynomial of degree n over $\mathbf{Q}$ with no multiple roots. Let C be a hyperelliptic curve defined by $y^2 = f(x)$. We denote by g the genus of C. We see that