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# INDUCING CHARACTERS OF PRIME POWER DEGREE

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## 1. Introduction

Let $G$ be a (finite) group and $\chi$ be an irreducible character for $G$. We consider the set of primitive characters that induce $\chi$. In general, there is very little that can be said about this set other than the degrees of these characters must divide $\chi(1)$. When $\chi(1)$ is a power of some prime, this set often has more structure. For example, if $p$ is an odd prime, $G$ is $p$–solvable, and $\chi$ is monomial with $\chi(1)$ a power of $p$, then every primitive character inducing $\chi$ must be linear (Theorem 10.1 of [7]). Given any prime $p$, a $p$–solvable group $G$ of $p$–length 1, and a character $\chi \in \mathrm{Irr}(G)$ where $\chi(1)$ is a power of $p$, it has been shown that every primitive character inducing $\chi$ has the same degree (Theorem A of [8]). It is easy to find examples of $p$–solvable groups that do not have $p$–length 1, but do have characters of prime power degree that are induced by primitive characters of different degrees. For example, $\mathrm{GL}_2(3)$ has a character of degree 4 that is induced by a linear character and a primitive character of degree 2. In [8], we construct an example where $p$ is odd. The purpose of this note is to prove that such examples cannot occur for characters of $p$–power degree where this degree is "small." With this in mind, we have the following theorem.

**Theorem A.** *Let $p$ be an odd prime, and let $G$ be a $p$–solvable group. Let $\chi \in \mathrm{Irr}(G)$ be a character of $p$–power degree less than or equal to $p^p$. Then every primitive character inducing $\chi$ has the same degree.*

Note that the monomial character of degree 4 in $\mathrm{GL}_2(3)$ that is also induced by a primitive character of degree 2 shows that Theorem A is not necessarily true when we do not assume that $p$ is odd. In [8], we find a $p$-solvable group that has character of degree $p^{p+1}$ that is induced by primitive characters of different degrees where $p$ is an odd prime. (The example in [8] has $p = 3$, but it is not difficult to find similar examples for many other primes.)

Using our methods, we also obtain an analogue to a result of Dade. The main theorem of [1] considers the following situation: $G$ is a $p$–solvable group for some odd prime $p$, the character $\chi \in \mathrm{Irr}(G)$ is monomial and has $p$–power degree, and $N$ is a subnormal subgroup. In this situation, he proved that if $\theta$ is an irreducible constituent of $\chi_N$, then $\theta$ is monomial. In other words, he proved that $\theta$ and $\chi$ are induced by