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SPECTRAL ANALYSIS OF AN ISOTROPIC STRATIFIED ELASTIC STRIP AND APPLICATIONS

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1. Introduction

The characteristics of an elastic isotropic stratified strip $\Omega = \{\mathbf{x} = (x_1, x_2); x_2 \in (0, L)\} \subset \mathbb{R}^2$ are the density $\boldsymbol{\rho}$ and Lamé coefficients $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ that we assume to be measurable functions, depending on x_2 only, and bounded from below and above by two positive constants. We shall derive a limiting absorption principle (LAP) and a division theorem for the selfadjoint operator $(\mathcal{D}(A), A)$ (see (2)) associated with Ω with Dirichlet and free surface conditions on $\{x_2 = L\}$ and $\{x_2 = 0\}$, respectively. These boundary conditions come from a model of a seismic problem. For other studies dealing with elasticity in different situations see for instance [7] and [12].

Roughly speaking, a LAP means that the resolvent operator $z \longrightarrow R_A(z) := (A - zI_d)^{-1}$ can be extended continuously to the essential spectrum (a part of the real axis) in suitable topologies. It is an important stage in scattering theory (cf. [1]). A division theorem enables to deal with a perturbed or a multistratified strip (cf. [3]).

A multiplication operator in $\oplus^\infty L^2(\mathbb{R}) := \{(f^n)_{n\geq 1}; \sum^\infty \|f^n\|^2_{L^2(\mathbb{R})} < \infty\}$ by a family of functions μ_n, $n \geq 1$, is defined by

$$
(1) \qquad \begin{cases} \mathcal{D}(M) &= \{(f^n)_{n\geq 1} \in \oplus^\infty L^2(\mathbb{R}); (\mu_n f^n)_{n\geq 1} \in \oplus^\infty L^2(\mathbb{R})\} \\ M(f^n)_{n\geq 1} &= (\mu_n f^n)_{n\geq 1}. \end{cases}
$$

As we will see A is unitarily equivalent to M with μ_n being the dispersion curves λ_n of A. Such a result holds for other differential operators, in particular for the acoustic operator B (cf. (9)) studied in [4] and [5]. But it is the first time that the following original phenomena are proved (cf. [3], ch.5 and ch.7). They do not take place in the acoustic case.

$$
(\mathcal{P}) \qquad \begin{cases} \bullet \text{ The functions } \lambda_n \text{ are not necessarily monotonic on } \mathbb{R}_+. \\ \bullet \text{ One can have } \lambda_n''(0) = \lambda_n'''(0) = 0 \text{ in addition to } \lambda_n'(0) = 0. \end{cases}
$$

One of our objectives is to show the spectral consequences of these phenomena and illustrate the fundamental difference between the elastic and the acoustic cases.