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KOLMOGOROV EQUATIONS IN HILBERT SPACES WITH APPLICATION TO ESSENTIAL SELF-ADJOINTNESS OF SYMMETRIC DIFFUSION OPERATORS

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1. Introduction

The essential self-adjointness of differential operators over infinite dimensional spaces has been extensively studied. For historical comments and literature on this topic see the monograph by Berezanskii [3] and a recent paper by Albeverio, Kondratiev and Röckner [2]. For some generalizations to certain Banach spaces we refer to Long [13]. Sometimes essential self-adjointness is also called strong uniqueness. There is another kind of uniqueness for symmetric diffusion operators, i.e. Markovian uniqueness, which means that one has uniqueness only within the class of self-adjoint operators which generate sub-Markovian semigroups. Obviously essential self-adjointness implies Markovian uniqueness. For details we refer to [2] and some references therein. In this paper, we aim to prove the essential self-adjointness of a certain class of perturbed Ornstein-Uhlenbeck operators associated to stochastic evolution equations (SEE) in a separable Hilbert space, by using a general parabolic criterion of Berezanskii [3]. In [4], Berezanskii and Samoilenko established the essential self-adjointness of Ornstein-Uhlenbeck operators with a certain potential perturbation by using the finite-dimensional approximation approach. In [17], Shigekawa proved the essential self-adjointness of perturbed Ornstein-Uhlenbeck operators by using Malliavin calculus. Our method is completely different from Shigekawa's. For our purpose, we need first to establish the existence and uniqueness of classical solutions to the Kolmogorov equations associated to the perturbed Ornstein-Uhlenbeck operators. The definition of classical solution will be given in Section 2 , following Cannarsa and Da Prato [5]. In [7], Da Prato proved the existence and regularity of classical solutions to Kolmogorov equations associated to Ornstein-Uhlenbeck operators.

We consider the semilinear SEE :

$$\begin{cases} dX(t) = [AX(t) + F(X(t))]dt + Q^{\frac{1}{2}}dW(t) \\ \qquad X(0) = x \in H \end{cases} \tag{1.1}$$

on a separable Hilbert space H with norm $\|\cdot\|$ and inner product $< \cdot, \cdot >$. We denote