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ALMOST QUATERNIONIC STRUCTURES ON EIGHT-MANIFOLDS

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1. Introduction

$Sp(n)$ is the group of the quaternionic linear automorphisms acting from the left on a right quaternionic n-dimensional vector space preserving a positive definite Hermitian form on it. $Sp(n)\cdot Sp(1)$ is the group $Sp(n)\times Sp(1)/\{(1,1),(-1,-1)\}$. If we identify $\mathbb{H}^n$ with $\mathbb{R}^{4n}$, the following left action on a right quaternionic n-dimensional space $\mathbb{H}^n$

$$(A,\alpha)v = Av\bar{\alpha}, \quad A\in Sp(n), \alpha\in Sp(1),$$

where $\bar{\alpha}$ is the quaternionic conjugate to α, induces an inclusion $Sp(n)\cdot Sp(1)\hookrightarrow SO(4n)$.

Let ξ be an oriented real vector bundle of dimension $4n$. We will say that ξ has an $Sp(n)\cdot Sp(1)$-structure iff its structure group $SO(4n)$ can be reduced to $Sp(n)\cdot Sp(1)$. Such a structure was treated i.e. in [2], [13], [16]. In the case of the tangent bundle of a smooth manifold it is common to talk about almost quaternionic structure. (See [1], [13].) The prototype of a manifold with such an almost quaternionic structure is the quaternionic projective space $\mathbb{H}P^n$. Examples of manifolds with almost quaternionic structure are quaternionic-Kähler manifolds whose holonomy group is by definition a subgroup of $Sp(n)\cdot Sp(1)$ ([1], [18], [13]).

This paper is devoted to $Sp(n)\cdot Sp(1)$ for $n=2$. (The case $n=1$ is not interesting since the group $Sp(1)\cdot Sp(1)$ is isomorphic to $SO(4)$.) Our aim is to find nontrivial sufficient and in some cases also necessary conditions for the existence of an $Sp(2)\cdot Sp(1)$-structure in oriented 8-dimensional vector bundles over oriented 8-manifolds in terms of characteristic classes and cohomology of the base manifold. Analogous results for the almost complex structure in dimensions 8 and 10 were obtained in [15] and [20]. One of the corollaries of our main results in Section 7 reads as

Theorem 1.1. *Let M be an oriented closed connected smooth manifold of*

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