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FLEXIBLE BOUNDARIES IN DEFORMATIONS OF HYPERBOLIC 3-MANIFOLDS

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0. Introduction

Let M be a cusped hyperbolic 3-manifold with non-empty geodesic boundary. A small Dehn filling deformation of M on the cusps can be performed so that the boundary is kept to be geodesic. Then assigning to each deformation a hyperbolic structure on the boundary, we get a map B_M from the space of such deformations to the Teichmüller space of ∂M. More precise argument for this fact will be given in §1.

Motivated by the conjectures posed in Cooper-Long [1] and Kapovich [5], Neumann and Reid [7] discovered many examples of M such that B_M is a constant map. Fujii [3] also obtained another concrete family of small deformations for some M such that B_M maps his family to a constant structure. These examples at first contrasted with our naive intuition that the deformation of hyperbolic structure affects everywhere. But the fact itself would not be too surprising once we realized that the dimension of the source can be bigger than that of the target and in that case B_M can never be injective. A more pertinent problem to set up for the moment would be what the map B_M looks like. In fact, we have known only a little about it so far.

Under the circumstances above, it would be worth finding examples for which we can convert this rather difficult problem to something we can do by hand. In this paper, we will construct infinitely many one-cusped examples of M so that B_M is a local embedding at the complete structure. The polyhedral construction will be discussed in §2. Then by using its polyhedral structure, we will compute the derivative of B_M at the complete structure by hand in the later sections.

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1. The map B_M

We briefly review Dehn filling deformations of cusped hyperbolic 3-manifolds. Let N be a noncompact, orientable, complete hyperbolic 3-manifold of finite volume, and $\bar{\rho}_0 : \pi_1(N) \to \mathrm{PSL}_2(\mathbf{C})$ its holonomy representation. According to