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Osaka J. Math.
33 (1996), 387–398

NON-LINEARIZABLE REAL ALGEBRAIC ACTIONS OF $O(2, R)$ ON R^4

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(Received March 18, 1994)

0. Introduction

In algebraic transformation groups, one of the important problems is the following.

Linearization problem ([6]). *Let G be a reductive complex algebraic group. Is any algebraic G action on affine space* $\boldsymbol{C}^n$ *linearizable, i.e. isomorphic to some G module as G variety?*

Some positive answers to this problem have been given (see [1] for a survey article) but in 1989, G.W. Schwarz [17] constructed counterexamples for many noncommutative groups with $O(2,\boldsymbol{C})$ being the most explicit case (in the case that the acting group is commutative, any counterexample have never found, and see [7], [9], [11], [12] for further recent results).

In this paper, we consider the analogous problem in the real algebraic category, which was posed in [15]. Then it would be appropriate to take a compact Lie group as acting group since there is a one-to-one correspondence between the family of compact Lie groups and that of reductive complex algebraic groups through the complexification (see [14] p.247).

Schwarz used the properties of complex algebraic geometry to find the counterexamples, so it is not clear whether his argument works in the real algebraic category because $\boldsymbol{R}$ is not algebraically closed. We use the methods of Masuda-Petrie [11] to obtain the following result.

Theorem. *There is a continuous family of algebraically inequivalent, non-linearizable real algebraic* $O(2,\boldsymbol{R})$ *actions on* $\boldsymbol{R}^4$.

Let G be a compact real algebraic group and G_C be the reductive complex algebraic group obtained from G via the complexification. Let $ACT(G,\boldsymbol{R}^n)$ (resp. $ACT(G_C,\boldsymbol{C}^n)$) be the set of equivalence classes of real algebraic G actions on $\boldsymbol{R}^n$ (resp. complex algebraic G_C actions on $\boldsymbol{C}^n$), where the equivalence relation is defined by G variety (resp. G_C variety) isomorphism. Then there is a complexification map