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HEAT KERNEL AND SINGULAR VARIATION OF DOMAINS

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1. Introduction

We consider a bounded region M in $\boldsymbol{R}^n$ ($n=2$ or 3) whose boundary is smooth. Let w be a fixed point in M. By $B(\varepsilon;w)$ we denote a ball of radius ε with the center w. We put $M_\varepsilon = M\backslash\overline{B(\varepsilon;w)}$.

Let $U(x,y,t)$ ($U^{(\varepsilon)}(x,y,t)$; respectively) be the heat kernel in M (M_ε ; respectively) with the Dirichlet condition on its boundary ∂M (∂M_ε; respectively). That is, it satisfies

$$(1.1)\qquad \begin{cases} (\partial_t - \Delta_x)U(x,y,t)=0 & x,y\in M,\quad t>0 \\ U(x,y,t)=0 & x\in\partial M,\quad y\in M,\quad t>0 \\ \lim_{t\to 0} U(x,y,t)=\delta(x-y) & x,y\in M \end{cases}$$

$$(1.1)\qquad \begin{cases} (\partial_t - \Delta_x)U^{(\varepsilon)}(x,y,t)=0 & x,y\in M_\varepsilon,\quad t>0 \\ U^{(\varepsilon)}(x,y,t)=0 & x\in\partial M_\varepsilon,\quad y\in M_\varepsilon,\quad t>0 \\ \lim_{t\to 0} U^{(\varepsilon)}(x,y,t)=\delta(x-y) & x,y\in M_\varepsilon \end{cases}$$

We put

$$(1.2)\qquad (U_t f)(x)=\int_M U(x,y,t)f(y)dy, \qquad f\in L^p(M)$$

and

$$(1.3)\qquad (U_t^{(\varepsilon)} f)(x)=\int_{M_\varepsilon} U^{(\varepsilon)}(x,y,t)f(y)dy, \qquad f\in L^p(M_\varepsilon).$$

Then, $U_t f$ and $U_t^{(\varepsilon)} f$ satisfy the following.

$$\begin{cases} (\partial_t - \Delta_x)(U_t f)(x)=0 \qquad x\in M,\quad t>0 \end{cases}$$