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DIFFUSION PROCESSES ON MANDALA

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1. Introduction

The concept of "fractal" is fairly broad. A mathematical framework of fractals were given by Hutchinson [5] (he call them *strictly self-similar sets*). His self-similar set K is a compact subset of a complete metric space X, and *invariant* with respect to a collection $\{F_1,\cdots,F_N\}$ of contraction maps on X: that is, $K=\cup_{i=1}^{N}F_i(K)$. One way to obtain the physical properties of these media is to construct Brownian motion on them. The study of diffusion processes on fractals was initiated by Kusuoka [11], Goldstein [3], and Barlow-Perkins [1]. They constructed Brownian motion on the Sierpinski gasket, and investigated it in detail. The Sierpinski gasket is one of Hutchinson's fractals of finitely ramified type (i.e., $\max_{i\neq j}\sharp(F_i(K)\cap F_j(K))<\infty$), which have been studied by many probabilists. Lindstrøm [13] introduced a class of finitely ramified fractals, containing the Sierpinski gasket, called "Nested fractals", and constructed Brownian motion on them. Kusuoka [12] and Fukushima [2] studied these processes by using (regular local) Dirichlet forms. Also, there are many works on nested fractals (for example see Shima [15] and Kumagai [10]). Post critically finite (P.C.F. for short) self-similar sets, a generalization of nested fractals, were introduced by Kigami [6], and he considered Laplace operators and Dirichlet forms on them.

Hutchinson's fractal (i.e., strictly self-similar set) K is associated with the full-shift symbolic space, i.e., there is a natural surjective map π: $\{1,\cdots,N\}^{N}\to K$, cf. Kigami [6]. In this paper, our object is a finitely ramified fractal which is not associated with full-shift, but with a Markov sub-shift. Let C be a unite circle in $\boldsymbol{R}^2$ having the origin as its center, and a collection $\{F_1,\cdots,F_5\}$ of 3-similitudes with fixed points (1,0), (0,1), $(-1,0)$, $(0,-1)$, (0,0), respectively. There exists a unique compact set $K\subset\boldsymbol{R}^2$ such that $K=\cup_{i=1}^{5}F_i(K)\cup C$, cf. Hata [4], which we call *the Mandala* (see Figure 1): this name is taken from the Buddhist magic diagram. However, we will be exclusively concerned with *the plain Mandala* which is a simplification of the Mandala, in order to avoid notational complications. The Mandala and the plain Mandala are not included in Hutchinson's framework.

We shall give a mathematical definition of the plain Mandala in Section 2. Our method of constructing diffusion processes is a modification of Kigami's method for P.C.F. selfsimilar sets [6] (see also Kumagai [9]). The plain Mandala is