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THE FUNDAMENTAL GROUP OF THE SMOOTH PART OF A LOG FANO VARIETY

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Introduction

Let X be a normal projective variety over the complex number field $\boldsymbol{C}$. We call X a *Fano variety* if X is $\boldsymbol{Q}$-Gorenstein and the anti-canonical divisor $-K_X$ is ample. A Fano variety X is said to be a *log Fano variety* if X has only log terminal singularities (cf. [6]). A Fano variety X is called a *canonical Fano variety* if X has only canonical singularities (cf. [6]). The *Cartier index* $c(X)$ is the smallest positive integer such that $c(X)K_X$ is a Cartier divisor. The *Fano index*, denoted by $r(X)$, is the largest positive rational number such that $-K_X \sim_{\boldsymbol{Q}} r(X)H$ ($\boldsymbol{Q}$-linear equivalence) for a Cartier divisor H.

This note consists of two sections. In §1, we shall consider canonical Fano 3-folds and prove the following:

Theorem 1. *Let X be a canonical Fano 3-fold. Let $X^o := X - \mathrm{Sing}\, X$ be the smooth part of X. Assume that X has only isolated singularities. Then we have*:
(1) *Suppose the Fano index $r(X)$ is* 1. *Then $\pi_1(X^o) = \boldsymbol{Z}/c(X)\boldsymbol{Z}$* (cf. Remark 1.1 in §1).
(2) *Suppose that the canonical divisor K_X is a Cartier divisor. Then X^o is simply connected.*

REMARK. (1) The assumption that X has only isolated singularities is used to prove Lemma 1.3 in §1.

(2) Using the same proof (see §1) one can show that $\pi_1(X^o) = \boldsymbol{Z}/c(X)\boldsymbol{Z}$ when X is a log Fano variety of Fano index one and with only isolated singularities because even in this case the $\boldsymbol{Z}/c(X)\boldsymbol{Z}$-covering Y constructed in §1 has only isolated canonical singularities.

In [19], we shall give a universal bound for $c(X)$. Under the much stronger condition that X has only terminal and cyclic quotient singularities, T. Sano