

THE ASYMPTOTIC EXPANSION OF THE FUNDAMENTAL SOLUTION FOR PARABOLIC INITIAL-BOUNDARY VALUE PROBLEMS AND ITS APPLICATION

Dedicated to Professor Hiroki Tanabe for his 60th birthday

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0. Introduction

Let M be a smooth compact Riemannian manifold of dimension n with smooth boundary Γ . In this paper we consider parabolic initial-boundary value problems as follow:

$$\left\{ \begin{array}{ll} \left(\frac{\partial}{\partial t} + P\right)u(t, x) = 0 & \text{in } (0, T) \times M, \\ Bu(t, x) = 0 & \text{on } (0, T) \times \Gamma, \\ u(0, x) = m(x) & \text{in } M, \end{array} \right.$$

where $P = -\Delta + h$ with a smooth vector field h on M of complex coefficients. The boundary operator B which we consider in this paper is related to one of the following conditions with smooth coefficients.

($\mathcal{D}$) the Dirichlet condition,

($\mathcal{N}$) the Neumann condition,

($\mathcal{R}$) the Robin's condition,

($\mathcal{O}$) the Oblique condition with parabolic condition, that is, $B = -\frac{\partial}{\partial n} +$

$b(x, D)$ with the outer unit normal vector field $\frac{\partial}{\partial n}$ and a vector field $b(x, D)$ satisfying (3.2) in §3

and

($\mathcal{S}$) the Singular boundary condition $B = -a(x)\frac{\partial}{\partial n} + b(x)$ with the following