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UNLINKING TWO COMPONENT LINKS

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1. Introduction

We define a *knot* to be a piecewise linear embedding of a circle, S^1, in either Euclidean 3-space, $\boldsymbol{R}^3$, or the 3-sphere, S^3. A *link* is defined to be the disjoint union of circles in $\boldsymbol{R}^3$ or S^3. A natural question to ask about a knot or link is: *how can this be untied*? Here by "untying" we mean, how many crossings need to be changed to transform our knot or link into a collection (with one element in the case of a knot) of trivial circles. Formally, we define the ***unknotting number***, $u(K)$, for a knot K (or the ***unlinking number*** for a link) to be the minimal number of crossing changes necessary to convert the diagram of K into a diagram of a trivial knot (link). This minimum is taken over all diagrams of the knot or link.

A list of unknotting numbers has been complied by Y. Nakanishi [8] for prime knots having 9 or fewer crossings. Of the 84 knots listed, the unknotting numbers of nearly one quarter were unknown in 1981. In the last decade, due to techniques by Lickorish [6], Kanenobu and Murakami [4] and others, the number of these small knots with unknown unknotting numbers has been reduced to about a half dozen. In this paper we provide a list of unlinking numbers for the "small" classical two component links. These are the prime, nonsplit links which have diagrams with 9 or fewer crossings.

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2. Four methods for determining $u(L)$

We begin with a link, L, with components A_L and B_L. Individually the components of L may be unknotted. In general, however, $u(A_L)\geq 0$ and $u(B_L)\geq 0$. We shall use $lk(A_L, B_L)$ to denote the ***linking number*** of A_L and B_L.

To determine the unlinking number of a link we need both upper and lower bounds. The upper bound is found experimentally by examining diagrams of the link. Generally, in our small links, we will see that a sharp upper bound can be found in a *minimal* diagram of the link. This is a diagram with the minimal number of crossings, where again this minimum is taken over all