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FREDHOLM DETERMINANT FOR PIECEWISE MONOTONIC TRANSFORNATIONS

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1. Introduction

Let F be a piecewise C^2 transformation from a finite union of bounded intervals I to itself. We assume

(A1) The lower Lyapunov number ξ is positive:

$$\xi = \liminf_{x\to\infty} \operatorname*{ess\,inf}_{n\in I} \frac{1}{n} \log|F^{n\prime}(x)| > 0 .$$

(A2) The mapping F is nondgenerate:

$$\operatorname*{ess\,inf}_{x\in I} |F'(x)| > 0 .$$

(A3) There exists a finite partition of I into subintervals, F is monotone on each of the subintervals, and the restrictions of F, F' and F'' to each of the subintervals can be extended continuously to its closure.

Here F^n stands for the n-th iterate of F:

$$F^n(x) = \begin{cases} F(F^{n-1}(x)) & n \geq 1 . \\ x & n = 0 , \end{cases}$$

In the present paper, we are concerned with the spectrum of the Perron-Frobenius operator P. The Perron-Frobenius operator P associated with F is originally a nonnegative contraction operator defined on L^1, the set of integrable functions, by

$$\int Pf(x)\, g(x)\, dx = \int f(x)\, g(F(x))\, dx ,$$

where g belongs to L^∞, the set of bounded measurable functions. The spectrum problem of P as an operator on L^1 is rather trivial: for instance, it is found in [14] that the unit disk is contained in the spectum of the Perron-Frobenius operator on L^1. Therefore, we will restrict P to BV, the set of functions with bounded variation. We consider that BV is a subspace of L^1 functions which admit versions with bounded variation. We define the norm on BV by