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NOTE ON ALMOST RELATIVE PROJECTIVES AND ALMOST RELATIVE INJECTIVES

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This paper is supplemental to [4], [6] and [7]. We shall show, under assumption of finite length, that when we study almost relative projectives, we may restrict ourselves only to certain special homomorphisms h in the definition of almost relative projectives [6] (see §1). In the similar manner to proof of the above fact, we shall give a criterion for an R-module M_0 to be almost M_1-projective, where R is a perfect ring and M_1 is an indecomposable R-module. We shall obtain, in §3, a generalization of [6], Theorem 1, where direct sums of local modules were studied. In this section we shall show the same property on direct sum of indecomposalbe modules. §§2 and 4 are the dual versions of §§1 and 3.

1. Almost relative simple-projectives

In this paper we always assume that R is a ring with identity and that every module is a unitary right R-module. Let M be an R-module. We denote *the socle, the Jacobson radical*, and *the length of* M by $\mathrm{Soc}(M)$, $\mathrm{J}(M)$ and $|M|$, respectively. If $\mathrm{End}_R(M)$ is a local ring, we say M is *an* LE *module*. We recall here the definition of almost relative projectives [6]. Let M and N be R-modules. For any diagram with row exact:

(1)
$$\begin{tikzcd}
M_1 \arrow[r, dotted, "\tilde{h}"] \arrow[d, "\oplus"'] & N \arrow[dl, dashed, "\tilde{h}"'] \arrow[d, "h"] & \\
M \arrow[r, "\nu"] & M/K \arrow[r] & 0
\end{tikzcd}$$

if there exists $\tilde{h}\colon N\to M$ with $\nu\tilde{h}=h$ or there exist a non-zero direct summand M_1 of M and $\tilde{h}\colon M_1\to N$ with $h\tilde{h}=\nu\,|\,M_1$, then N is called *almost M-projective*. (if we obtain only the first case, we say that N is *M-projective* [2]).

Here we shall introduce a little weaker condition than the above. In the diagram (1) we take only the $h'\colon N\to M/K$ whose image is simple. If for any h' above there exists $\tilde{h}$ in the definition, then we say N is *almost M-simple-*