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ALMOST IDENTICAL IMITATIONS OF (3, 1)-DIMENSIONAL MANIFOLD PAIRS AND THE BRANCHED COVERINGS

AKIO KAWAUCHI

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0. Introduction

By a *good* (3, 1)-*manifold pair* (M, L) (or a *good* 1-*manifold* L *in a* 3-*manifold* M), we mean that M is a compact connected oriented 3-manifold and L is a compact proper smooth 1-submanifold of M such that any 2-sphere component of the boundary ∂M meets L with at least three points. For a compact connected oriented 3-manifold E, let $\partial_0 E$ be the union of all tori in ∂E and $\partial_1 E = \partial E - \partial_0 E$. Let $\mathrm{int}\, E = E - \partial E$ and $\mathrm{int}_0\, E = E - \partial_0 E$. A compact connected oriented 3-manifold E is said to be *hyperbolic* if $\mathrm{int}\, E$ (when $\partial_1 E = \emptyset$) or the double $\mathrm{D}(\mathrm{int}_0\, E)$ pasting along $\partial_1 E$ (when $\partial_1 E \neq \emptyset$) has a complete Riemannian structure of constant curvature -1. Then we define the *volume* $\mathrm{Vol}\, E$ of E to be the hyperbolic volume $\mathrm{Vol}(\mathrm{int}\, E)$ (when $\partial_1 E = \emptyset$) or the half hyperbolic volume $\mathrm{Vol}(\mathrm{D}(\mathrm{int}_0\, E))/2$ (when $\partial_1 E \neq \emptyset$), and the *isometry group* $\mathrm{Isom}\, E$ of E to be the hyperbolic isometry group $\mathrm{Isom}(\mathrm{int}\, E)$ (when $\partial_1 E = \emptyset$) or the quotient by τ of the following subgroup $\{f \in \mathrm{Isom}(\mathrm{D}(\mathrm{int}_0\, E)) \mid f\tau = \tau f\}$ (when $\partial_1 E \neq \emptyset$), where τ denotes the unique isometry of $\mathrm{D}(\mathrm{int}_0\, E)$ induced from the involution of $\mathrm{D}(\mathrm{int}_0\, E)$ interchanging the two copies of $\mathrm{int}_0\, E$ (cf. [22]). By Mostow rigidity theorem (cf. [23], [24]), $\mathrm{Vol}\, E$ is a topological invariant of E and $\mathrm{Isom}\, E$ is a unique (up to conjugations) finite subgroup of the diffeomorphism group $\mathrm{Diff}\, E$. Furthermore, there is a natural isomorphism $\mathrm{Isom}\, E \cong \mathrm{Out}\, \pi_1(E) = \mathrm{Aut}\, \pi_1(E)/\mathrm{Inn}\, \pi_1(E)$ and for any finite subgroup G of $\mathrm{Diff}\, E$ there is a natural monomorphism $G \to \mathrm{Out}\, \pi_1(E)$, so that G is isomorphic to a subgrpup of $\mathrm{Isom}\, E$. In a previous paper [8], for each good (3,1)-manifold pair (M, L), we have constructed an infinite family of almost identical imitations (M, L^*) of (M, L) such that the exterior $E(L^*, M)$ of L^* in M is hyperbolic. In this paper, we shall strengthen this result from the viewpoint of regular branched coverings.*)

DEFINITION: A good (3,1)-manifold pair (M, L) has the *hyperbolic covering property* if for any component unions L_0, L_1 (possibly, $\emptyset$) of L with $L_1 = L - L_0$,

*) By coverings, we will mean connected coverings.