Kawanaka, N.
Osaka J. Math.
28 (1991), 759–791

ON SUBFIELD SYMMETRIC SPACES OVER A FINITE FIELD*

Dedicated to Professor Nobuhiko Tatsuuma on his sixtieth birthday

NORIAKI KAWANAKA

(Received October 9, 1990)

1. Introduction

Let $\boldsymbol{G}$ be a connected reductive linear algebraic group defined over a finite field $\boldsymbol{F}_{q'}$. We put

$$G=\boldsymbol{G}(\boldsymbol{F}_q)\,, \quad q=q'^2\,.$$

Then the q'-th power Frobenius map σ: $\boldsymbol{G}\to\boldsymbol{G}$ induces on G an involutory automorphism τ: $g\to{}^\tau g$ with the fixed point set

$$G_\tau=\boldsymbol{G}(\boldsymbol{F}_{q'})\,.$$

We are concerned with the irreducible representations of the Hecke algebra $H(G, G_\tau)$, or, almost equivalently, with the zonal spherical functions on the *subfield symmetric space* G/G_τ. (A similar object, in the category of real Lie groups, is also being studied; see, e.g., [9], [24].) In the present paper, we take up the following problem:

(A) Classify the irreducible representations of $H(G, G_\tau)$, and determine their dimensions.

Since the classification of the irreducible representations of G is well-understood by works of G. Lusztig (see [21]), we can reduce problem (A) to the following one:

(A′) For each irreducible character χ of G determine the multiplicity $m_\tau(\chi)=\langle 1_{G_\tau}^G, \chi\rangle$ with which χ appears in the induced character $1_{G_\tau}^G$.

For an irreducible character χ of G, let $c_\tau(\chi)$ be the *twisted Frobenius-Schur indicator* [13]:

$$c_\tau(\chi)=|G|^{-1}\sum_{g\in G}\chi({}^\tau gg)\,.$$

*This work was supported by Grant-in-Aid for Scientific Research, The Ministry of Education, Science and Culture.