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# Q-HOMOLOGY PLANES WITH $C^{**}$-FIBRATIONS

Dedicated to Professor Heisuke Hironaka on his sixtieth birthday

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**Introduction.** Let $X$ be a nonsingular algebraic surface defined over the complex number field $\boldsymbol{C}$. We call $X$ a *homology plane* (resp. a $\boldsymbol{Q}$-*homology plane*) if the homology groups $H_i(X; \boldsymbol{Z})$ (resp. $H_i(X; \boldsymbol{Q})$) vanish for $1 \leq i \leq 4$. We can also define a *logarithmic homology plane* $X$ as a normal affine surface which has only quotient singularities and $H_i(X; \boldsymbol{Z})=0$ for all $i>0$.

In our previous paper [7], $\boldsymbol{Q}$-homology planes with Kodaira dimension less than 2 are classified and it is shown that there are many $\boldsymbol{Q}$-homology planes which have non-trivial automorphisms of finite order. A structure theorem is given on logarithmic homology planes of Kodaira dimension $-\infty$ and 1. In particular, it is proved that a logarithmic homology plane of Kodaira dimension $-\infty$ is isomorphic to one of the following surfaces:

(1) $\boldsymbol{C}^2$;

(2) $\boldsymbol{C}^2/G$, where $G$ is a small finite subgroup of $GL(2, \boldsymbol{C})$;

(3) A surface $X$ with an $\boldsymbol{A}^1$-fibration $\rho: X \rightarrow \boldsymbol{A}^1$ such that every fiber is irreducible and that there are $N$ multiple fibres $H_1, \cdots, H_N$ with respective multiplicities $d_1, \cdots, d_N$, each of them carrying a cyclic quotient singular point of type $d_i/e_i$, where $N$ is an arbitrary positive integer.

Similarly, logarithmic homology planes of Kodaira dimension 1 are studied by making use of $\boldsymbol{C}^*$-fibrations.

In the present paper we are interested in homology planes with $\kappa=2$. An example of a contractible algebraic surface with $\kappa=2$, which is a special case of a homology plane, was first given by C.P. Ramanujam [9] and many examples were recently found by Gurjar-Miyanishi [2], Miyanishi-Sugie [6] and Petrie-tom Dieck [11, 12]. We constructed in [6] homology planes by the blowing-up method from the configurations of two curves on the projective plane $\boldsymbol{P}^2$ and Petrie-tom Dieck [11] from the line arrangements on $\boldsymbol{P}^2$. In order to construct further examples, we propose to think of algebraic surfaces with fibrations of curves. As a natural extension of the $\boldsymbol{C}$-fibrations and the $\boldsymbol{C}^*$-fibrations which are so effective in the cases of $\kappa=-\infty$ and 1, we shall look into a surface with