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ALMOST IDENTICAL IMITATIONS OF (3, 1)-DIMENSIONAL MANIFOLD PAIRS

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Dedicated to Professor Fujitsugu Hosokawa on his 60th birthday

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By a 3-*manifold* M, *we mean a compact connected oriented* 3-*manifold throughout this paper.* Let $\partial_0 M$ be the union of torus components of ∂M and $\partial_1 M = \partial M - \partial_0 M$. In the case that $\partial_1 M = \emptyset$, if Int M has a complete Riemannian structure with constant curvature -1 and with finite volume, then we say that M is *hyperbolic* and we denote its volume by Vol M. Next we consider the case that $\partial_1 M \neq \emptyset$. Then the double, $D_1 M$, of M pasting two copies of M along $\partial_1 M$ has $\partial_1 D_1 M = \emptyset$. If $D_1 M$ is hyperbolic in the sense stated above, then we say that M is *hyperbolic* and we define the *volume*, Vol M, of this M by Vol M = Vol $D_1 M/2$. In this latter case, M is usually said to be *hyperbolic with* $\partial_1 M$ *tatally geodesic* (cf. [**T-1**]), but we use this simple terminology throughout this paper. When M is hyperbolic, ∂M has no 2-sphere components and by Mostow rigidity theorem (cf. [**T-2**], [**T-3**]), Vol M is a topological invariant of M. By a 1-*manifold in* M, we mean a compact smooth 1-submanifold L of M with $\partial L = L \cap \partial M$ and the pair (M, L) is simply called a (3,1)-*manifold pair*. A 1-manifold L in M is called a *link* if $\partial L = \emptyset$, a *tangle* if L has no loop components, and a *good* 1-manifold if $|L \cap S^2| \geq 3$ for any 2-sphere component S^2 of ∂M. A (3,1)-manifold pair (M, L) is also said to be *good* if L is a good 1-manifold in M. In [**Kw-1**], we defined the notions of imitation, pure imitation and normal imitation for any general manifold pair. In Section 1 we shall define a notion which we call an *almost identical imitation* (M, L^*) of (M, L), for any good (3,1)-manifold pair (M, L). Roughly speaking, this imitation is a normal imitation with a special property that if q: $(M, L^*) \to (M, L)$ is the imitaiton map, then $q|(M, L^* - a^*)$: $(M, L^* - a^*) \to (M, L - a)$ is ∂-relatively homotopic[1] to a diffeomorphism for any connected components a^*, a of L^*, L with $qa^* = a$. Let P be a polyhedron in a 3-manifold M. For a regular neighborhood N_P of P in M (meeting ∂M regularly), the diffeomorphism type of $E(P, M) = \mathrm{cl}_M (M - N_P)$ is uniquely determined by the topological type of the

[1] This homotopy can be taken as a one-parameter family of normal imitation maps.