Kawata S.
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MODULE CORRESPONDENCE IN AUSLANDER-REITEN QUIVERS FOR FINITE GROUPS

SHIGETO KAWATA

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1. Introduction

Let G be a finite group and k be a field of characteristic $p>0$. Let Θ be a connected component of the stable Auslander-Reiten quiver $\Gamma_s(kG)$ of the group algebra kG and set $V(\Theta)=\{vx(M)\,|\,M$ is an indecomposable kG-module in $\Theta\}$, where $vx(M)$ denotes the vertex of M. As we shall see in Proposition 3.2 below, if Q is a minimal element in $V(\Theta)$, then $Q\leq_G H$ for all $H\in V(\Theta)$. In particular we see that Q is uniquely determined up to conjugation in G.

Let $N=N_G(Q)$ and let f be the Green correspondence with respect to (G, Q, N). Choose an indecomposable kG-module M_0 in Θ with Q its vertex. Let Δ be the connected component of $\Gamma_s(kN)$ containing fM_0. The purpose of this paper is to show that there is a subquiver Λ of Δ and a graph isomorphism $\psi\colon \Lambda\to\Theta$ such that ψ^{-1} behaves like the Green correspondence f as a bijective map between modules in Λ and those in Θ. In particular Θ is isomorphic with a subquiver of Δ. Also it will be shown that if $H\in V(\Theta)$, then $H\leq_G N_G(Q)$.

The notation is almost standard. All the modules considered here are finite dimensional over k. We write $W\,|\,W'$ for kG-modules W and W', if W is isomorphic to a direct summand of W'. For an indecomposable non-projective kG-module M, we write $\mathcal{A}(M)$ to denote the Auslander-Reiten sequence terminating at M. A sequence $M_0-M_1-\cdots-M_t$ of indecomposable kG-modules M_i $(0\leq i\leq t)$ is said to be a *walk* if there exists either an irreducible map from M_i to M_{i+1} or an irreducible map from M_{i+1} to M_i for $0\leq i\leq t-1$. Concerning some basic facts and terminologies used here, we refer to [1], [5], [6] and [8].

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2. Preliminaries

To begin with, we recall some basic facts on relative projectivity.

Let H be a subrgoup of G and $\{g_i\}_{i=1}^{n}$ be a right transversal of H in G. If W and W' are kG-modules, then $(W, W')^H$ denotes the k-space $\mathrm{Hom}_{kH}(W, W')$.