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EXAMPLES ON POLYNOMIAL INVARIANTS OF KNOTS AND LINKS II

Dedicated to Professor F. Hosokawa on his 60th birthday

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Since the discovery of the Jones polynomial in 1984, several polynomial invariants of the isotopy type of knots and links in a 3-sphere have been discovered. In general, the relationships among them, together with the classical Alexander polynomial, are as follows: the (many variable) Alexander polynomial specializes to the reduced Alexander polynomial, the 2-variable Jones polynomial, which is a skein invariant, specializes to both the reduced Alexander and the Jones polynomials, and the Kauffman polynomial specializes to both the Jones and the Q polynomials. Remember [17, Fig. 4]. For a 3-braid knot or link, the 2-variable Jones and the Q polynomials are determined by the reduced Alexander polynomial and the exponent sum [10, 22]. This is generalized to a formula for the 2-variable Jones polynomial [21]. For a 2-bridge knot or link, the Q polynomial is determined by the Jones polynomial [14]. The purpose of this paper is to consider the independency of the polynomial invariants of the 2-bridge knots and links and the closed 3-braids.

In the previous paper [13], the following examples for the 2-bridge knots and links are constructed: arbitrarily many 2-bridge knots with the same Jones polynomial, arbitrarily many skein equivalent 2-bridge links with the same 2-variable Alexander polynomial, and a pair of skein equivalent 2-bridge links with distinct 2-variable Alexander polynomials. In Sect. 3, we construct: arbitrarily many skein equivalent fibered 2-bridge knots (Theorem 1), arbitrarily many skein equivalent 2-bridge links with mutually distinct 2-variable Alexander polynomials (Theorem 2), and arbitrarily many 2-bridge links with the same 2-variable Alexander polynomial but mutually distinct Jones polynomials (Theorem 3).

In Sect. 4, we construct the following examples concerning the Kauffman polynomial of the 2-bridge knots and links: a pair of skein equivalent 2-bridge knots with the same Kauffman polynomial (Theorem 4), a pair of 2-bridge knots with the same Kauffman polynomial but distinct Alexander polynomials (Theorem 5), a pair of skein equivalent 2-bridge links with the same Kauffman and 2-variable Alexander polynomials (Theorem 6), and a pair of skein equivalent