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NON-HOMOGENEOUS KÄHLER-EINSTEIN METRICS ON COMPACT COMPLEX MANIFOLDS II

Dedicated to Professor Shingo Murakami on his sixtieth birthday

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In the previous paper K-S [12] we have considered $P^1(C)$-bundles over compact Kähler-Einstein manifolds to obtain non-homogeneous Kähler-Einstein manifolds with positive Ricci tensor. The purpose of this paper is to give more examples of non-homogeneous compact Kähler-Einstein manifolds, more precisely, compact almost homogeneous Kähler-Einstein manifolds with disconnected exceptional set. By [1] and [8], the structure of orbits of almost homogeneous projective algebraic manifolds with disconnected exceptional set have been investigated, but no explicit examples were given in [1] and [8] except complex projective spaces. To construct these examples, we start again with $P^1(C)$-bundles over Kähler C-spaces and consider compact complex manifolds obtained from these $P^1(C)$-bundles by blowing down. Note that compact complex manifolds obtained from projective algebraic manifolds by blowing down are not Kähler in general as an example of Moisezon [14] Chap. 3, section 3 shows. We construct our compact complex manifolds in section 3 and prove that our compact almost homogeneous complex manifolds are Kähler and have positive first Chern class (Theorem 4.1). But in general these almost homogeneous manifolds may be homogeneous. We give a sufficient condition for these Kähler manifolds being non-homogeneous (Theorem 5.1). In section 6 we show that for each positive integer d there are compact Kähler-Einstein manifolds which have cohomogeneity d. We follow the notation in Kobayashi-Nomizu [11] which is slightly different from the one in [12].

1 Kähler C-spaces and Dynkin diagrams

We recall known facts on compact simply connected homogeneous Kähler manifolds, called Kähler C-spaces (cf. Takeuchi [18]).

Let Π be a Dynkin diagram and Π_0 a subdiagram of Π. The pair (Π, Π_0) is said to be *effective* if Π_0 does not contain any irreducible component of Π. Let Σ be the root system with the fundamental root system Π. Choose a lexicographic order $>$ on Σ such that the set of simple roots with respect to $>$ coincides with Π. Take a compact semi-simple Lie algebra $\mathfrak{g}_u$ with the root